method
TEACHING OF MATHEMATICS

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Prof. B. C. Rai

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Method
Teaching of Mathematics

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By

B.C

Prof. ~~C. B.~~ RAI

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PUBLISHER'S NOTE

Mathematics occupies an important place in the curriculum of the study of the higher secondary classes. For the students (male) of High School classes, it is a compulsory subject. It is, therefore, natural for it to be given an important place in the curriculum of the training colleges.

For the students of the training colleges, *i. e., B. Ed., L. T., B. T. Classes*, there is dearth of time for the study of the theory portion of their curriculum. The present book is intended to meet their need of the study of the method of the teaching of Mathematics.

Since the students of the training colleges have less time to study theory, they require the standard material on the theory of the methods of teaching of different subjects in the form in which they have to produce it at their final examination. The present book has been written, with a view to meet this requirement. Standard material on the teaching of Mathematics has been collected at one place, and presented in question-answer form, a method suitable for the students. Every point has been illustrated with the help of the examples. In order to illustrate the theory, *certain lesson plans have been given at the end of the book. A chapter dealing with the new Mathematics*, that has been introduced at the higher secondary stage of education, has also been included in this book. Every attempt has been made to make the book useful for the teachers under training.

In preparation of the book, the author has drawn from the works of different authors of the subject and periodicals and journals. The author and the publishers are grateful to all such authors and their publishers.

Suggestions for the improvement of the book shall be gratefully acknowledged.

Prakashan Kendra,
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CHAPTER I

AIMS AND OBJECTS OF THE TEACHING OF MATHEMATICS

Q. Describe briefly the aims and objects of the teaching of Mathematics ?

Or

What are the aims and objects of the teaching of Mathematics ? Discuss briefly.

Answer. Importance of Mathematics.—Mathematics has a practical value in life. Mathematics is called the practical cultural Science. We can neither know things correctly nor can we have practical utility of calculation, unless we have the knowledge of Mathematics. Apart from the personal life, Mathematics has an importance in social life as well. In society certain evaluations have to be done. These evaluations cannot be done properly unless help is taken from Mathematics through "permutations" and combinations chiefly. Mathematics has, therefore, a lot of utility for life. Mathematics does not have a practical value only. It has a cultural value, a disciplinary value and so on and so forth. It, therefore, occupies an important place in curriculum and in society.

Importance of aims and objects.—No subject can be taught without having aims and objects of the teaching of that subject clearly before the eyes. These aims and objects are the guiding star. They provide proper direction to the teacher and the students. These aims and objects are in accordance with the aims and objects of the society. The aim of education is to teach the students to be able to play their role as able citizens in the society. The aims and objects of the teaching of a subject should, therefore, be guided by this very consideration. This is true about Mathematics as well.

Aims and objects of the teaching of Mathematics.—Aims or the goals or the values that are kept in mind while doing a particular thing. Objectives or objects are the means to achieve those aims. One aim has, therefore, several objectives attached to it. It is these objectives that are helpful in determining the values and the aims of the teaching of Mathematics.

Values and aims are more or less the same. It is these values that serve as aims of the teaching of Mathematics. Educationists, guided by these values, get a place for Mathematics in the curriculum. Following are the aims of Mathematics :—

(1) Cultural aim.

(2) Disciplinary aim.

(3) Practical or utilitarian aim.

Cultural aim of the teaching of Mathematics.—Educationists have now come to realise that Mathematics has a lot of cultural value. It helps formation of certain habits in the students. It also helps the students to grow in cultured citizens. Culture is not possible unless there is proper development of the power of reasoning and judgment. It is Mathematics that brings about the development of these qualities along with the qualities of concentration, thinking, preciseness etc. The aim of the teaching of Mathematics is to develop cultured citizens who can discharge their obligations to the society effectively and successfully.

Disciplinary aim of education.—Education is no aim in itself. It is a means to an end, the end being the proper building up of the society. Education brings about the development of habits, mental qualities, efficiency and such other things in the students. These things are helpful in discipline.

Simplicity, accuracy, certainty of results, originality, reasoning and co-relation of the teaching of the subject with the problems of life are the chief characteristics of discipline. Mathematics is quite helpful in the development of these characteristics and qualities, It brings about simplicity, develops accuracy and certainty of results encourages the students to have original thinking and proper reasoning and also helps them to co-relate the problems of Mathematics with the problems of every-day life.

Practical or utilitarian aim of the teaching of Mathematics.—Mathematics, apart from the value of bread and butter, has a value of wider practical application. Principles and theories of Mathematics are applied in different aspects of life. Thinking and reasoning are very much guided by Mathematics, and it is thinking and reasoning that help the building of the society.

Mathematics is helpful in other branches of Sciences such as engineering, physics, economics, acturial science, etc. No business can flourish and grow without practical knowledge of Mathematics. The teaching of Mathematics is, therefore, guided by this utilitarian aim.

Objectives of the teaching of Mathematics.—Aims have certain objectives attached to them. No aim can be achieved without the help of these objectives. In order to achieve a particular aim, the teacher has to resort to certain objectives. While teaching certain topic, he has to keep some objective before him. It is these objectives that are tested at the examination. This is true of all tests, whether written or oral. Following are the objectives of the teaching of Mathematics :—

1. **Development of abilities.**—Mathematics keeps in view, the following objectives in regard to development of the abilities of the students :—

(*a*) To bring about the development of the ability of the students to express their thoughts clearly, accurately and precisely.

(*b*) To enable the students to systematically organise and interpret the data given.

(*c*) To help the students to reach correct conclusions common to all by taking recourse to accurate and logical reasoning.

(*d*) To enable the students to grow the capacity analyse a problem, discovering fundamental relationship.

(*e*) To help the students to develop the power of original thinking and investigation.

(*f*) To develop in the students the capacity to exercise intuitive powers and common sense.

(*g*) To enable the students to accurately generalise special concepts.

2. **Development of the power of appreciation.**—In regard to the objective of the development of the power of appreciation, the following things are kept in mind :—

(*a*) The contribution of Mathematics to physical and natural sciences, engineering, philosophy, etc.

(*b*) The influence that Mathematics exercises upon human progress and civilization.

(*c*) The vocational value of Mathematics in modern business and industrial activities.

(*d*) Mathematics is a mode of thought that serves as a model for scientific thinking in other fields. In other words, Mathematics helps the students to have scientific thinking in various fields of life.

(e) Exact processing and accurate results are important requisites of individual as well as social life. Mathematics is helpful in the attainment of these objectives leaving the important ones.

(f) There are certain cultural values of Mathematics. It enables the students to have proper utilisation of leisure besides developing their mental faculties.

Objectives of the teaching of Mathematics in regard to development of correct and proper attitudes.—One of the important objectives of the teaching of Mathematics is the development of proper and correct attitudes. In this regard, the following are aimed at :—

(a) Formation and development of the habits of systematical and logically pursuing a task to completion.

(b) Cultivation and development of the proper habits of study and power of concentration.

(c) To train the students in scientific thinking and reasoning.

(d) To help the students to attain the power for accurate and effective expression.

(e) To enable the students to develop the ability of independent and original thinking.

(f) To encourage the students to acquire knowledge with an open mind and for the sake of its utility in life and also for the sake of acquiring knowledge.

(g) To build in the students the qualities of self-confidence and such other abilities that help the building up of a strong personality.

(h) **To develop the capacity of utilising the subject in day-to-day life.**—Mathematics has a value in our day-to-day life. Students should be taught to utilise the subject properly and this can be done by training them to measure length, weight, capacity, etc. This is one of the objectives of the teaching of the Mathematics.

(i) **To develop the capacity of analysing a problem.**—Students should be taught to develop the capacity of successful analysis of the problem that confronts them. This helps the youngmen in life and, therefore, it would be worthwhile to keep it as an objective of the teaching of Mathematics.

(j) **Formation of behaviour pattern.**—Formation of behaviour pattern or change in the behaviour pattern is an objective of the teaching of Mathematics. It is through behaviour pattern that it is possible for a student to understand a problem and to know the subject through which he can reach the results.

(*k*) **Application of knowledge.**—We have already talked about the objectives of the application of Mathematics in day-to-day life. Objective of the application of knowledge is nothing but the application of the subject to day-to-day life.

(*l*) **Knowledge of assumptions.**—In Mathematics, for the solution of cerain problems, certain assumptions have to be made. The knowledge of these assumptions is not clear to the students. The objective of the teaching of Mathematics should be to make these assumptions clear and precise to the students.

(*m*) **Generalisation.**—From a particular knowledge, they should be taught to have a generalised knowledge.

(*n*) **Knowledge of teachnical terms.**—In Mathematics there are a number of technical terms. Implications of these technical terms and their connotations have to be made clear to the students. This must be one of the objectives of the teachings of Mathematics.

Q. Describe briefly the values of the teaching of Mathematics. ?

Answer. Values of the teaching of Mathematics.—Values are quite important in the teaching of a subject. It is these values that help the educationists to determine the objectives of the teaching of a particular subject. Let us now talk briefly in the following lines the value of the teaching of Mathematics.

Practical value of Mathematics·—Modern civilization owes a lot to Science and Science owes a lot to Mathematics. The impact of Mathematics is well evident on every branch of Science, whether it is Engineering, Psycology, Biology, Physics or Chemistry.

The practical application of Science in many fields cannot be done without the help and assistance of Mathematics.

In a day-to-day life, every person, whether he is a coller or a tailor, a carpenter, a masion, a clerk or an intellectual, has to apply Mathematics for his livelihood as well as for coming out to exact results and conclusions. It not only helps people to adjust their income and expenditure but it also helps them to know things in the proper perspective.

Value of Mathematical knowledge to the individual.—Individuals also have a lot to gain from Mathematics. There shall be very few students who shall have nothing to do with Mathematics in their practical life. Those who go in for the job of office assistants may not have to apply the principles of Mathematics in their noting and drafting, but while in maintaining the account of their files and other schemes, they shall have to apply the principles of Mathematics. Similarly, surveyors, engineers and others too shall have to apply this subject.

Value of Mathematics in regard to outlook.—Mathematics is very helpful in developing an exact outlook. Generally it is said that a person who has studied Mathematices has an accurate or exact outlook. This is an indirect value of Mathematics but very useful in developing useful citizens for the society.

Informative value of Mathematics.—Mathematics provides a lot of information to the students who study it. This information is helpful not only in regard to Mathematics but in regard to other branches of knowledge as well, such as History, Literature, Geography, etc. In fact, knowledge of Mathematics suits every intelligent mind. In the begining it was studied as a knowledge for the sake of knowledge. Now no subject is studied with that object in view but Mathematics may be said to have the value in this regard as well.

Cultural value of Mathematics.—We have already seen that Mathematics is helpful in the study of the development of the nature and other developments that continually go on, around us. Development of civilization and culture is possible through calculation. *Prof. J. W. A. Young* has rightly said :—

> "Wherever we turn in these days of iron, steel and electricity we find that Mathematics has been the pioneer. Were its backbone removed, our material civilization would inevitably collapse. Modern thought and belief would have been altogether different, had Mathematics not made the various sciences exact. Different laws of Science and the scientific instruments are based on exact mathematical concepts. Astronomy and the Physics are the most exact sciences and the exactness is the result of the usefulness of Mathematics".

The disciplinary value of Mathematics.—Mathematics has a disciplinary value. It disciplines the mind and develops the reasoning power. *Lock* was of the opinion that Mathematics is a way to settle in the mind a habit of resoning. A person who has studied Mathematics is capable of using his power of reasoning in an independent and easy manner. He can take to thinking more effectively. All this helps him to acquire knowledge properly.

Simplicity of reasoning.—Mathematics simplifies the reasoning. It helps the students to grasp difinite facts and arrive at the right results.

Accuracy of reasoning.—In order to arrive at accurate results, the reasoning has to be accurate and exact. It is the Mathematics that helps in this regard.

Certainty of results.—Solution to a problem is either correct or incorrect. Personal factor cannot come in Mathematics as it is possible to apply to it in social sciences. Element of subjectivity has no place in Mathematics. All these things help the certainty of resutls.

Originality of thinking.—The learning of this subject provides opportunity for original thinking. Solution to many of the problems are found out only with the help of original thinking.

Solution of the problems of life due to similarity.—Problems of day-to-day life are similar to the problems solved in Mathematics. Teaching of Mathematics is, therefore, helpful in the solution of problems of day-to-day life.

Verification of results.- Through Mathematics it is possible to have verifiication of the results.

Concentration of mind.—Mathematics is helpful in developing the concentration of mind. This habit is helpful in other spheres as well. Clarity and exactness of thinking are necessary in day-to-day life as well as Mathematics. Mathematics teaches people about these things.

Development of constructive imagination and inventive faculities.—Solution of mathematical problems is nothing but making inventions and discoveries. If a student is able to solve certain problems of Mathematics, he can also make inventions and discoveries for future life.

Development of correct reasoning.—We have already discussed this aspect of the teaching of the Mathematics. In fact, development of Mathematical reasoning enables the students to stand on their own and apply their own faculties. They grow confident about their own powers and are able to judge their own work. This will all help the youngmen to face life correctly and boldly.

Building up of character.—Exactness, mental discipline, power of reasoning, etc. are helpful in the development of character. A person who passesses all these qualities will be called a man of character and will be able to face life more successfully. Mathematics is also helpful in the stimulation of will power, truthfulness, honesty, patience, devotion etc. These qualities are helpful for proper development of character and life.

Precision of expression.—Exact thinking always leads to exact expression. Mathematics is also helpful in the development of exact and accurate expression. Mathematics is, therefore, also helpful in the development of precise language, which is free from superfluous terminology and redundant words.

POINTS FOR REVISION

(1) Mathematics has a lot of utility for life. Along with the practical value, it has cultural, disciplinary and other values as well.

(2) Like the aims and objects of teaching of other subjects, teaching of Mathematics too has its aims and objects.

(3) Following are the aims and objects of the teaching of Mathematics :—

- (*a*) Cultural Aims :
- (*b*) Disciplinary Aims ; and
- (*c*) Practical Aim of the teaching of Mathematics.

(4) Along with the aims of the teaching of Mathematics, following are the different objectives of the teaching of Mathematics :—

- (*a*) Development of the abilities, such as the ability to organise thoughts and present them systematically.
- (*b*) Development of the power of appreciation.
- (*c*) Development of correct and proper attitudes such as concentration, thinking, reasoning, etc.
- (*d*) Development of the capacity to utilise the subject in day-to-day life.
- (*e*) Development of the capacity of analysing a problem.
- (*f*) Formation of behaviour pattern.
- (*g*) Application of the knowledge of the subject in day-to-day life.

(*h*) Knowledge of assumptions.

(*i*) Generalisation.

(*j*) Knowledge of technical terms.

(5) The teaching of Mathematics has certain values as well. They are given below :—

(*a*) Practical value of Mathematics.

(*b*) Value of mathematical knowledge to the individual.

(*c*) Value of Mathematics in regard to outlook.

(*d*) Informal value of Mathematics.

(*e*) Cultural value of Mathematics.

(*f*) Disciplinary value of Mathematics.

(*g*) Simplicity of reasoning.

(*h*) Accuracy of reasoning.

(*i*) Certainty of results.

(*j*) Originality of thinking.

(*k*) Solution of the problems of life due to similarity.

(*l*) Verification of results.

(*m*) Concentration of attention and mind.

(*n*) Development of constructive imagination and inventive faculties.

(*o*) Development of correct reasoning and precision of expression.

(*p*) Building up of character.

CHAPTER II

PLACE OF MATHEMATICS IN CURRICULUM

Q. Describe briefly the causes of the importance of Mathematics in the curriculum? Should Mathematics be made an optional subject at High School stage? Give reasons for and against it.

Answer. Important place of Mathematics in the curriculum.—Mathematics occupies an important place in the curriculum. This has been there since ages. Plato was of the view that those persons who are not able to study Mathematics and understand its problems should not be allowed to enter an institution. Almost all eminent educationists of past as well as of present, including Herbert, Froebel, Pestalozzi, Dr. Maria, Montessori, Sir T. P. Nuna, etc. have unequivocally laid down the importance of the Mathematics. They are of the view that without the study of Mathematics, the intellectual and cultural development of a person is not possible. While studying Mathematics, invariably the words of Napolean the Great 'without development of Mathematics no nation can develop' continue to ring in our ears.

All these things go to prove that Mathematics has an important place in the curriculum. This is so on account of its practical utility in life. Let us now try to talk the factors that have given this subject an important place :—

Utility of Mathematics.—We have already talked about the values of Mathematics. Under these values, we have seen the utility of Mathematics in life. It is an undeniable fact that Mathematics has an important place of utility in life In business, in offices, in industry and everywhere the work is conducted on scientific lines, Mathematics has a practical role to play.

Contribution to different branches of knowledge.—Almost all the branches of knowledge that have some element of calculation have to depend on Mathematics. Not only natural sciences, such as Physics, Chemistry, etc. but social sciences, such as Economics, Political Science, Social Science, etc., also owe a lot to Mathematics. In short, no calculation is possible without taking recourse to Mathematics.

Utility of Mathematics in Natural Sciences.—Natural Sciences draw a lot from Mathematics. Physics cannot lay down in formula about steam engine without taking support from Mathematics. This is true of other natural sciences as well.

Contribution to Arts and Crafts.—Mathematics makes and has made a valuable contribution to arts and crafts. Arts and crafts acquire practical utility only, which is possible through Mathematics only. It is not possible to have engineering in its proper form without the help of Mathematics. This is true of other branches of technical know-how and technology.

Mathematics and education.—In education or in the actual process of education, Mathematics has an important role to play. Progress of Education cannot be determined without employing the techniques of Mathematics. When we come to psychology, it is difficult to find out intelligence quotient, co-efficient of correlation, etc. without taking the help from Mathematics.

Contribution to democracy and citizenship.—With the growing needs of democracy and citizenship, Mathematics has come to occupy an important place. Calculation of votes and exactitudes etc. are the contribution of Mathematics. So in a democratic society Mathematics has a greater utility.

Importance in day-to-day life.—Mathematics occupies an important place in our day-to-day life. No work of society can be conducted without the application of Mathematics. Even in our recreation, we employ the principles of Mathematics.

Place of Mathematics in the curriculum.—In the curriculum, Mathematics occupies an important place. In our country, in most of the States Mathematics is taught as a compulsory subject upto the higher secondary or secondary stage of education. Secondary education is the minimum requirement for life. After completing this education, some students enter into life while others go in for higher education. It is on account of this fact that it has been suggested that the curriculum at the secondary stage should be terminus as well as preparatory and should comprise of Mathematics.

In certain States of India Mathematics is a compulsory subject up to Junior High School or Middle School standard only. This is quite scientific. After Middle School eduction or Junior High School education, students go in for technical education or education in training institutes and polytechnics. Others take up other aspects of life. Education in these places would not be complete without some knowledge of Mathematics.

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Mathematics as a compulsory subject for boys.—Generarlly Mathematics is a compulsory subject for boys. Girls have the option to leave this subject after Junior High School education or Middle School education. This is correct in the sense that there is differentiation of curriculum for the girls. Girls do not have to take up job in life so much as the boys have. Knowledge of Mathematics or Arithmetic up to Junior High School is sufficient for household work and that is why it is considered sufficient for them.

Mathematics as an optional subject in higher education.—In higher education Mathematics is not a compulsory subject. Only those students study Mathematics who have a desire to pursue some specialised vocation or profession. This is correct in the sense that everybody is not expected to apply higher Mathematics in every-day life.

Justification for the place of Mathematics in curriculum.—The discussion given above justifies the place given to the Mathematics in the curriculum. No other subject is so important in life as Mathematics. Knowledge of Mathematics of a particular standard is useful for our day-to-day life. The standard of the knowledge of Mathematics that is required in day-to-day life of an average nature and that is why Mathematics of that standard is invariably taught as a compulsory subject in High Schools for boys.

Mathematics as optional subject at High School stage

There has been a long discussion between the two schools of thought, one who believe that Mathematics should be made optional at the high school stage and the other who argue that it should be a compulsory one. The ideas and arguments put forward by the two schools of thought seem to be equally sound. The first school suggests that every body after his high school education is not going to be a scientist, or an engineer or any body to whom the facts of mathematics are necessary for future work. Moreover they argue that it should be optional at the high school stage because in most of the democratic countries like America and England it is optional. Thirdly, they argue that as in other countries a man who has learnt mathematics up to middle classes is capable of earning his living, it is also possible in our country. The arguments put forward by these gentlemen appear to be sound. But they do not see to the other side of the picture. Can they or any body else prophecy or judge whether who is going to be an engineer or a mechanic ? If we cannot judge, it is our duty to provide to the student a broad view of what may lie in the future and give him a better course to enable him to judge for himself as to his best line of endeavour and not to make him regret his early choice. It is an admitted fact that mathematics appears to be rather a dull subject but it may be said that it is not dull but you and your teacher who made it dull Mathematics is an exact science and its facts do not go out-of-date; they remain ever as they were before. If many of the students fail in mathematics at the high school stage, the fault does not lie with the subject but with the teacher who is not able to create in the students a zeal for the subject. Making it an optional subject will bring out the fact that to a large number of those who do not elect to make mathematics during their school course, the choice of a vocation will become very narrower. It is essential that every educated man must have sufficient knowledge of that kind of mathematics which will make him understand and appreciate the important and increasing part played by mathematics in the progress of modern civilisation and society. If you shut out mathematics from a pupil's school course, the door of almost every form of useful occupation and almost of every form of higher studies is closed for him. Education for him becomes very narrow indeed.

The argument that it should be optional because it is in foreign countries, is also baseless. The terms 'mathematics' and 'High School' do not have the same meaning that we, in India attach to it. The high school student in those countries corresponds to the Intermediate student of our country and the mathematics that they want to make optional in their school is much higher both in quality and quantity. Moreover the average mental age of the high school boys in foreign countries is at least two years higher than the average mental age in our schools.

We can make this subject optional at the high school stage only if we have the same standard of the students as in those countries. It can be made optional if our students at the middle school age are made acquainted with all the necessary mathematics they require for their daily life so that they do not feel their ignorance of mathematics as a great handicap in their progress.

POINTS FOR REVISION

(1) Every subject gets a place in the curriculum on the basis of its utility in life and education. This is true of Mathematics as well.

(2) Mathematics occupies an important place in the curriculum because of the following factors :—

(*a*) It has a utility in life.

(*b*) It has contributed to different branches of knowledge.

(*c*) It has a lot of utility in the study of natural sciences.

(d) It has contributed a lot to arts and crafts.

(e) Education also owes a lot to Mathematics. This is true to educational statistics very much.

(f) It has contributed towards democracy and citizenship.

(g) It has important place in day-to-day life

(3) On account of the facts enumerated above, Mathematics occupies an important place in the curriculum of different stages of education.

(4) Mathematics is a compulsory subject for the boys up to High School standard.

(5) In the Intermediate, it is an optional subject and the same is true about higher education as well.

(6) It is perfectly justified to give an important place to the Mathematics in the curriculum.

(7) There is a view that Mathematics should be made an optional subject at High School stage and some Educational Boards have done so, but it is a mistake.

Chapter III

THE CURRICULUM OF MATHEMATICS

Q. What is the importance of curriculum in the teaching of a subject? Why is it essential for a teacher to study curriculum?

Or

What do you mean by curriculum? What are the various principles of the organisation of the curriculum materials?

Or

Critically examine the curriculum of the Junior High School classes, keeping in view the principles enumerated above?

Answer. Need for the planning of curriculum.—Curriculum is the content material or the material for the study of the subject-matter. It is an integral part of the educational process. In Mathemetics also curriculum occupies an important place. It is the material for study. The curriculum has to be planned and organised in a scientific manner, keeping in view the psycho-physical requirements of the students. If the curriculum is properly organised, it enables the students to know the subject-matter that they have to study. On the other hand, it also makes clear to the teacher, the material that they have to teach to the students. It also makes it clear to the examiners.

Stages of the planning or construction of curriculum.—Curriculum has to be, as already stated, constructed and planned in a scientific manner. It has to undergo the following two stages :—

(*a*) Selection of the curriculum ; and

(*b*) Organisation of the curriculum material.

While constructing the curriculum, the aims and objects of the teaching of a particular subject have to be kept in mind. Let us now examine these things a bit in detail.

Principles of the selection of the curriculum or subject-material.—Selection of the curriculum material cannot be carried out in a haphazard manner. It has to be properly planned and to be on certain principles. These principles are being described briefly in the following lines :—

1. The cultural and the disciplinary value.—Teaching of Mathematics has cultural and disciplinary value. No curriculum can be constructed without keeping in view these values. The main object of education is to develop useful habits in the students and to enable them to discharge the duties successfully as citizens. The selection of the material of the curriculum of Mathematics has to be guided by these principles. If the subject-matter has been properly selected, it helps the teacher as well as the taught. The teacher is built to train the mind of the students properly and the students are able to feel at home while they are studying.

2. The material selected should be preparatory as well as terminus.—The material of the curriculum should be so selected that it should be preparatory as well as terminus. On the one hand, it should train the students for life and, on the other, it should also prepare them for higher education. After every stage of education, some students enter life while others go in for higher education. The curriculum should be able to provide satisfactory education to both these categories of the students.

3. Principle of utility or use.—No doubt, knowledge for the sake of knowledge is a good aim, but in the present circumstances it is not a successful aim. Utilitarian aspect of every subject-matter has to be kept in mind. Unless a particular subject-matter has use in it, it shall not be able to keep the interest of the students. From this consideration, the curriculum material of the subject should have the following ingredients in it :—

1. It should be useful for day-to-day life.
2. It should be useful for the study of other subjects.
3. It should have some vocational utility in it. In other words, it means that there should be a common ground for a fairly good number of vocations.
4. It should enable the students to understand and appreciate the role played by Mathematics in the development of civilization and other branches of knowledge.

Views of J. W. A. Young in regard to selection of the subject-matter of curriculum.—According to Young, selection of the subject-matter of curriculum should be guided by the following considerations or principles :—

"1. To exhibit most clearly and to best advantages the Mathematical type of thought.

2. To help to a better understanding of the laws of nature.

3. To bring out distinctly the mathematical relationships that exist in the social organism and in the activities of modern life, and to show how Mathematics aids in solving their problems.

4. To give sufficient skill in the actual performance of mathematical processes to meet the future needs of the pupil.

5. To permit the organisation of the material into a homogenous whole meeting the demands of the scientific pedagogy."

Curriculum material for recreational Mathematics.—Puzzles, Crosswords, etc. are the subject-matter of the Mathematics. If the students are taught Mathematics through recreation, they shall be able to learn the subject-matter in a playway method. This, on the one hand, has a recreational value as well as educative value. This principle has to be kept in view more in reference to the students of the younger age group, or lower classes.

Organisation of the curriculum.—Organisation of the curriculum lies in distribution of the subject-matter of the curriculum in different classes. It is also very necessary and has to be planned in a scientific manner. It should not be left to people who have very little to do with the actual job of the teaching. Teachers, who are the field workers in education, should be given an opportunity to lay down the principles of organisation. They should also be given an opportunity to associate themselves actively with the organisation of the subject material of the curriculum.

Underlying principles of the organisation of the curriculum.—Curriculum has to be organised on the basis of certain principles. It is these principles that help the scientific planning of the curriculum. The planning has to be done keeping in view the objectives of the teaching of the subject and the objectives of the society as well as the requirements of the pupils. Generally following principles are kept in mind while organising the curriculum :—

(*a*) **Psychological bias.**—The organising of the curriculum should be done on the principles of psychology. It means that the subject should be so organised that it would create interest in the students and motivate the learning. It should also keep in view various psychological requirements of the students.

(*b*) **Logical order.**—The subject should be organised in a logical order. It means that the subject-matter that is organised for class V should be such that it is not repeated in class VI or it has not already been taught in class IV. It also means that the organisation of the subject-matter of Mathematics should be so organised that the teaching of a topic should precede or succeed a topic in its logical sequence or order.

(*c*) **Principle of activity.**—Activity today plays an important part in the teaching of a subject-matter. These activities are a part of the ordinary life. Mathematics is used in different phases of the activities. Following activities should be kept before the eyes while organising the subject-matter of the curriculum|:—

(1) Personal and home activities.

(2) Recreational activities.

(3) Vocational activities.

(4) Community, civic and social activities.

(5) National activities.

(*d*) **Practical use.**—Today education lays a good deal of importance to the principle of 'learning by doing'. Practical activity or practical work should, therefore, keep in mind the practical use. In other words, it means that while organising the subject-matter of Mathematics practicals should be given an important place.

(*e*) **Difficulty.**—Organisation of the content material of the curriculum should keep in view the mental development and the capacities of the students. Various topics of the syllabus and the curriculum have to be so organised that they prove to be useful for the future citizens of the nation. These topics cannot be useful for the students of all the age groups. Certain topics are useful for the students of a particular age group while the other topics are useful for the students of other age groups. While organising the content material of the curriculum, this fact has to be kept in mind. If it is kept in mind, there shall be lesser difficulties.

Topical versus spiral method of arrangement.—Some educationists are of the opinion that the curriculum should be organised through topical method. It means that a particular topic should be started in a particular class and finished over there. This method has not been approved by all the educationists. They say that if this method is applied, many irrelevant things may creep in and many relevant things shall be left out. The students may have to study things that are not of immediate importance to them. It is also said that if this method is adopted, very few topics shall find place in a particular class, because all sorts of questions on a particular topic shall have to be solved by the students in one class.

Such educationists prefer the *Spiral Method* of the organisation of the curriculum. In this method a topic or the subject is divided into a number of smaller independent units to be dealt with. . . . . that suits the mental capacities of the pupils. They argue that this method is more natural and less tiring to the students. In support of this method, they put forward the argument that in this method the student loses nothing in accuracy and gains a lot in the power of application of the general rules to particular problems.

Individual versus class.—The general prevalent view is that the needs and the requirements of the whole group of students that forms a class should have priority over the individual requirements of the students. Now educationists do not agree with this view. They are of the view that the subject-matter of the curriculum should be so organised that the differences in the abilities of the individual children are also kept in mind. Capacities of the individual student should not be neglected. While keeping in view the requirements of the whole class, the individual differences of the pupils in intelligence, interest, experiences, maturity, etc. should also be kept in mind. The curriculum should provide for homogeneous grouping or ability grouping of the students as well as diagnostic and remedial work. It should also provide for individual attention to the students.

Principle of correlation.—The principle of correlation has now come to stay in the field of education. No subject can now be taught in isolation. Every subject is related to the other in one way or the other. The subject-matter of the Mathematics should be organised, keeping in view the principle on correlation. Correlation may be of different varieties. While organising the content material of the Mathematics, the following types of correlation must be kept in mind :—

1. Correlation of Mathematics with the problems of everyday life.

2. Correlation of Mathematics so far as possible, with the other subjects of the syllabus or curriculum.

3. Correlation between the different branches of Mathematics.

4. Correlation of different topics of a particular branch of Mathematics.

Other requirements of the organisation of the curriculum of Mathematics.—Apart from the principles enumerated above, the organisation of the curriculum of Mathematics should also be done, keeping in view the following things :—

1. **Voice of the teacher.**—No curriculum can be complete unless it has the voice of the teacher. The teacher is the actual field worker and his experiences must help the construction of the curriculum. His specialised knowledge is very useful.

2. **Child-centred syllabus.**—The curriculum should be child centred and not subject or topic centred. Unless the requirements of the child are kept in view, the curriculum shall not be useful.

3. **From imperical to rational.**—The curriculum should be so organised that the students are able to acquire facts through experience in tuition and induction. They should also be gradually made to feel the need for and appreciate the usefulness of deduction system of education. This method is known as the method of proceeding from imperical to rational. In other words, it means that the students should be made to proceed from known to unknown or from general to particular.

4. **From concrete to abstract.**—The curriculum should provide for the teaching of the subject on the principle of providing from concrete to abstract. Students should, first of all, be presented from the experiences and the objects that are concrete and from there they should be made to proceed to abstract principles.

Place for organised teaching and no place for incidental teaching.—Incidental teaching is now not considered scientific. Incidental teaching is not useful for life. A student may know about a particular thing without having any relationship with the topic that is to follow or that has already proceeded. Such a teaching will not be useful. Teaching should, therefore, be organised on the lines proposed. The curriculum of Mathematics should provide for such a teaching.

Problems.—While organising curriculum, due place should be given to the problems. Problems should be linked with every topic and subject. These problems should have bearing not only on the subject but also on life.

Curriculum at different stages of education.—Normally, different stages of education are divided as follows :—

1. Pre-primary.
2. Primary.

3. Junior High School or Middle School.
4. Secondary.
5. University education or higher education.

Curriculum of Mathematics for different stages of education has to be constructed, keeping in view the requirements the students as well as the society. If an attempt is made to give a model curriculum, it shall not be wise, because curriculum of different states of India as well as other countries differ from place to place. However, in the next chapter, an attempt shall be made to give a brief outline of the aims, objects, syllabii and teaching aids at different stages of education.

POINTS FOR REVISION

(1) Curriculum of every subject has to be planned on the basis of certain principles and objectives. This is true of Mathematics as well.

(2) The curriculum of different stages of Mathematics has to be planned on principles suited to the stage.

(3) There are two stages of the planning or contruction of the curriculum :—

(*a*) Selection ; and

(*b*) Organisation.

(4) The following principles govern the selection of the curriculum or material of the subject of Mathematics :—

(*a*) The cultural and the disciplinary value.

(*b*) The material selected should be preparatory as well as terminus.

(*c*) The principle of utility or use should guide the selection of material for the curriculum.

(*d*) The famous educationist, Young, has laid down that the subject-matter of the curriculum should train the educands to acquire certain skills that shall be useful for their practical life.

(*e*) There should also be provision for recreational material for the curriculum of Mathematics.

(5) The organisation of the curriculum of Mathematics should be guided by the following :—

(*a*) Psychological bias.
(*b*) Logical order.
(*c*) Principle of activity.
(*d*) Practical use or utility.
(*e*) Difficulty.

(6) The arrangement of the subject-matter should be done on the combined basis of spiral method as well as topical method.

(7) There should be proper opportunity for individual differences, while keeping in view the requirements of the class.

(8) The principle of co-relation should invariably be kept in mind while organising the curriculum.

(9) Apart from the requisites enumerated above, the following things should also be kept in mind in the organisation of the curriculum of Mathematics :—

(*a*) Voice of the teacher should play a vital part in the organization.
(*b*) The syllabus should be child-centred.
(*c*) The subject-matter should proceed from known to unknown.
(*d*) It should proceed from concrete to abstract.

(10) In the organization of the curriculum, there should be provision for organised teaching and no place for incidental teaching.

(11) While organising curriculum, problems should invariably be set.

(12) There should be difference in the curriculum of the following stages of education :—

(*a*) Pre-primary.
(*b*) Primary.
(*c*) Junior High School or Middle School.
(*d*) Secondary.
(*e*) University or higher education.

(13) While organising curriculum of different stages of education, the psycho-physical and other requirements of the students must be kept in mind.

(14) Hence Mathematics is necessary for human beings at eary step without the knowledge of mathematics a man can not move about. All the subjects of School are dependant upon Mathematics. Hence the Mathematics should find a respectable place in School curriculum.

CHAPTER IV

CORRELATION IN THE TEACHING OF MATHEMATICS

Or

CORRELATION OF MATHEMATICS WITH OTHER SCHOOL SUBJECTS

Q. Describe briefly the correlation of Mathematics with other school subjects ?

Or

"Every subject has to take help from Mathematics and so it has a bearing on other subjects." Discuss this statement.

Or

Describe briefly the relation of Mathematics with Science, Geography, Drawing, History, Language, etc. ?

Answer. Significance of correlation.—Today it is believed that entire knowledge is one. It has been divided into different compartments, in order to bring about efficiency and convenience in the study. We find that there are different subjects such as Mathematics, Science, Geography, History, etc. They are treated as different branches of knowledge by some educationists. Often we [teach these subjects, we teach them as if they are independent subjects. This is not a correct attitude. Today it is believed that all the subjects are correlated. The knowledge of one subject is, therefore, important in a manner that is has a bearing on other subjects and the bearing on other subjects is also taken into account while teaching a particular subject.

Significance of correlation in the teaching of Mathematics.—Teaching of Mathematics is done, keeping in view its correlation with other subjects. It is quite significant for the teaching of this subject as well. It facilitates the understanding of the subject-matter by the students. It is also possible to correlate different experiences and, at the same time, allow different aptitudes and inclinations to work with co-ordination and correlation. The mind of the student works in such a manner that it is possible for him to understand a subject that is taught, keeping in view its correlation with other subjects. Such a teaching also brings about healthy development of the personality. Sometimes when the teacher as well as the taught finds that there are so many subjects in the syllabus, he gets perturbed and feels difficulty in studying. If all the subjects are taught in correlation, this difficulty is easily solved.

Kinds of correlation.—Correlation of every subject is of three kind :—

1. Correlation between difierent branches of the subject.

2. Correlation between different topics of a particular branch or subject.

3. Correlation of one subject with other subjects. This correlation is of two types :—

(*a*) Regular and systematic ; and

(*b*) Subsidiary or indirect.

All these kinds of correlation are found in Mathematics as well. In the following lines, an attempt shall be made to study these different kinds of curriculum of the Mathematics as a subject.

Correlation between different branches of Mathematics.—Mathematics is not one subjedct. It is, in fact, a combination of certain subjects, namely, Arithmetic, Algebra, etc. Apart from these branches of Mathematics, there is Higher Mathe matics, which includes Statistics, Hydro-dynamic, Mechanics, etc. In fact, all these branches of Mathematicts are correlated. In a simple manner, it can be said that several difficult questions of Arithmetic may be solved through the equations and principles of Algebra. On the other hand, L. C. M., G. C. M., Fractions, Square root etc. in Algebra are based upon the rules of Arithmetic. That is true about Algebra as well. While finding out the area of a particular triangle or a quardrangle, the principles of Arithmetic are applied. The basic rules of Arithmetic such as addition, substraction, square root etc. are found in all branches of Arithmetic.

Higher mathematics has a lot of bearing upon the elementary rules and regulations of mathematics. Unless a student of a higher mathematics is well aware of the rules and regulations of Mathematics, he shall not be able to solve the problems of higher Mathematics. In short, it may be said that different branches of Mathematics such as Geometry, Algebra, Hydrodynamics, Statics and Dynamics etc. are very much correlated. While teaching Mathematics, it is desirable to make it clear to the students that this correlation shall be helpful in their future study. Correlation with the topic in hand should also be made clear to them. If this correlation is made clear, they shall benefit by it presently as well as at future date.

Correlation between different topics of a particular chapter or branch of Mathematics.—What is true about different branches of Mathematics is also true about different topics of a chapter of Mathematics. A particular chapter of Artihmetic, say L. C. M., has the relationship with the rules of addition and substraction. Similarly, in Algebra, while solving an equation, the principle of simplification has to be taken help of. The same is true of Geometry. Unless a student is clear about the rules of drawing a triangle, he cannot draw a rectangle and divide it into two triangles.

While teaching Mathematics, the teacher should try to keep the principle in mind that different topics|of a particular chapter are correlated. He should try to clarify this position as well. This would facilitate the study of the subject and also help the students to make use of the subject at a future date.

Correlation of Mathematics with other subjects.—Mathematics cannot be taught in isolation. If it is done, the purpose of the education shall be defeated. The correlation of one subject with other subjects is of two types :—

(*a*) Direct ; and

(*b*) Indirect.

Direct correlation of Mathematics with other subjects.—In this process, the teacher organises his lesson plan in such a manner that he is able to clarify the correlation of certain aspsects of Mathematics with certain other aspects of other subjects, such as Physis, Chemistry, etc. He can very safely explain the correlation by explaining the use of the aprinciple in other subjects. In Phys certain calculations are don with the help of Mathematics. The teacher can very safely explein that aspect of Physics in which the help from Mathematics is taken.

Indirect correlation of Mathematics with other subjects and aspects of life.—This is said to be more psychological and interesting method. In this subject, an attempt is made to give reference of certain other subjects while teaching a particular topic or subject of Mathematics. Such a reference makes the teaching interesting. Invariably the teacher should try to refer the things that are already known to the students. This would, on the one hand, facilitate understanding of the topic that is going on in the class room and on the other, it would also help the students to acquire a broad and comprehensive outlook.

Correlation of Mathematics with certain other subjects.—Mathematics is a subject, which has more of bearing on subjects of of pure sciences, such as Physics, Chemistry, etc. It has a bearing on social sciences as well. In the following lines, an attempt has been made to study the correlation of Mathematics with different subjects.

Mathematics and Physics.—Mathematics is rather inevitable for Physcis. In Physics a lot of calculation is done. Really speaking, Physics is fifty per cent of Mathematics. Without calculation, no formula of Physics can come out correct. If it has been possible for men to conquer space, it has been possible only to do it through calculation. The principle of evaporation ·is based on certain calculation. The power is generated only on the basis of the principles of Mathematics. In short, it may be said that Mathematics has a strong correlationship with Physics and neophysics and even the metaphysics.

Mathematics and Chemistry. What is true of Physics is, to a very great extent, true of Chemistry as well. Many of the principles of Chemistry are based on calculations. No experiment can be conducted in Chemistry without taking help from the balance, weighing and such other things that are based on principles of Mathematics. In short, it can be said that Mathematics and Chemistry are closely related.

Mathematics and Biology.—Biology consists of Zoology and Botany. Directly speaking, these two subjects have very little to do with the principles of Mathematics, but indirectly they also owe a lot to Mathematics. The graphs and charts, that are drawn in Zoology as well as in Botany, have in them the basic principles of Mathematics.

Mathematics and Science.—Science is a very wide subject. It includes different branches of the pure sciences. It has a good deal to do with Mathematics. Graphs and charts, curves, etc., all are based on the principles of Mathematics. In Astronomy and Hydromechanics it is the principles of Mathematics that work. In short, it may be said that Science and Mathematics are correlated.

Mathematics and Social Sciences.—The relationship between Mathematics and social sciences cannot be said to be very direct and very strong. However, social sciences draw a lot on the principles of Mathematics. Social sciences have charts, graphs, curves, etc. All these things are based on the principles of Mathematics.

Today statistics are very important for social sciences. Without these statistics it is not possible to have a correct study of any subject. It is Mathematics that provides these statistics. In fact, statistics is nothing but an integral part of Mathematics. In the following lines we shall try to assess the correlation between different social sciences and Mathematics.

Mathematics and Geography.—Geography, as we all know, is a study of the physical conditions of a particular country or society. In this subject we study about rivers, canals, mountains, population, etc. Attempt is also made to measure the height and draw graphs and charts. In all these things, it is the Mathematics that plays its role. Relief maps are based purely on Mathematics. Geology, which is considered to be a separate branch of science, has a good deal of bearing on Geography. Geology tries to study the surface of the earth and the underneath surface of the earth. The formation of the earth and other things are also studied in it. While studying all these things, al ot of calculation is to be done and then charts and figures are to be drawn. The longitude and latitude and the calculations of time are studied in Geography. These things can be studied only with the help of the principles of Mathematics and Geometry.

Mathematics and Economics.—Economics is a social science that studies the earning and spending factors of a society. Population, production, distribution, etc. are studied under it. In Economics statistics play a very important role. Economical problems of population cannot be studied properly unless correct statistics are available. It is here that Mathematics has its importance.

In Economics also, charts, graphs and such other things are drawn. In this regard also the Mathematics plays its role.

Mathematics and History.—The relationship between History and Mathematics is reciprocal. History provides help to Mathematics in knowing about different mathematicians who have contributed a lot to the field of knowledge. It is History that provides knowledge about the origin and development of Mathematics. On the other hand, in regard to calculation of dates etc. it is the Mathematics that helps History.

Drawing and Mathematics.—Drawing has different branches, such as geometrical drawing, memory drawing, figure drawing, etc. In Geometrical drawing, it is the principles of Geometry that are applied. In this regard, the relationship is quite intimate and strong. On the other hand, the sketching of the different figures of geometry depends upon drawing. This is true of field book as well. In engineering, it is the combination of these two subjects that helps. In short, it may be said that the relationship between drawing and Mathematics is quite strong. The exactness of a figure can be measured only with the help of Mathematics. It is, therefore, advised that in class room, while teaching Mathematics, drawing should be taken help of and also correlationship, if possible, established.

Language and Mathematics.—No subject can be taught without the help of language. This is true of Mathematics as well. The principles of Mathematics are expressed through the medium of language. In this respect, it is the language that helps Mathematics. If the language that is spoken in a Mathematics class or written in a book is not correct, the exact idea of the subject-matter shall not be conveyed. This shall have a wrong impression in the mind of the students.

Apart from the help rendered by language to Mathematics, there is the story of the Mathematics helping the language as well. Teaching of Mathematics develops an attitude of exactitude in the students. This exactitude helps them to write the language correctly. In short, it may be said that the relationship between the two is quite significant.

Teaching of Mathematics with other aspects of life.—Mathematics cannot be taught in isolation. The utility of this subject is not confined to the class room or the school only. It has a bearing on life. As far as possible, while teaching Mathematics, a reference should be made to its use in actual life. While we go out for shopping and purchase certain things, we get them measured and weighed. In this activity it is the Mathematics that plays its part. The students should, therefore, be explained the utility of Mathematics in practical life.

The teacher of Mathematics should also try to show to the students the practical application of the principles of Mathematics. He may also show the big electrical installations and bridges etc. and explain to the students that all this has been possible only with the help of Mathematics. This would impress upon them the utility of the subject-matter in life and the correlation between actual life and Mathematics. This would be scientific as well as an interesting method.

POINTS FOR REVISION

(1) Correlation has a lot of significance in the modern life. The principle of correlation has to be observed in education as well.

(2) The principle of correlation has also to be observed and maintained in the teaching of Mathematics.

(3) Correlation of a subject is of the following kinds :—

- (*a*) Correlation between different branches of the subjects.
- (*b*) Correlation between different topics of a particular branch or subject.
- (*c*) Correlation of one subject with other subject. This correlation is of the following two kinds :—
  1. Regular and systematic.
  2. Subsidiary or indirect.

In the Mathematics we find the correlation of both these types.

(4) We find that there is correlation in different branches of Mathematics such as Arithmetic, Algebra, Geometry, Trignometry, etc.

(5) There is also correlation between different topics of these different branches of Mathematics with the exception of most languages.

(6) There is direct correlation between Mathematics and Physics. They draw from each other inspiration.

(7) What is true of Physics is also true of Chemistry. It means that there is a very near relationship between Mathematics and Chemistry.

(8) Relationship between Mathematics and Biology is not very direct and very close. However, the maps, the charts and graphs of Biology have the basic principles of Mathematics in them.

(9) Apart from the branches of Science enumerated above, Mathematics has its relationship with other branches of science also.

(10) The relationship of Mathematics with social science is also important. Social sciences make use of the Mathematics in the data and the statistics.

(11) Geography and Mathematics are related. In the calculation, it is the Mathematics that comes to the help of Geography. On the other hand, Geography also provides some knowledge about the development of Mathematics in different countries of the world.

(12) Mathematics has its relationship with Economics as well and specially the problems of population. Several theories of Economics are based on the principle of Mathematics.

(13) The relationship between History and Mathematics is reciprocal. If History gives us knowledge about the contribution of different persons to the field of Mathematics, Mathematics also helps History in calculations in giving exact dates of the happenings.

(14) Mathematics and Drawing are closely related. Geometrical Drawing, which is a branch of Drawing, at large draws a lot from Geometry, a branch of Mathematics. On the other hand, in the Drawing of the charts, figures, etc. .it is the Drawing that helps the students of Mathematics. In the field of engineering, it is the combination of the two that is very much to be seen.

(15) The relationship between language and Mathematics is of a general nature. No subject can be taught without the use of Language. This is true of Mathematics as well. On the other hand, Mathematics helps the students of the Language to develop an exact attitude towards life and other things.

(16) Apart from these subjects, Mathematics has relationship with other aspects of life as well. The teacher of Mathematics should try to look at all these aspects and help the students to realise and visualise them.

CHAPTER V

PRINCIPLES OF THE TEACHING OF MATHEMATICS

Q. Describe briefly the principles that govern the teaching of Mathematics ?

Answer. Importance of principles.—No education is complete unless it is guided by certain principles. These principles form the basis of the teaching. This is true of almost all the subjects. Mathematics is no exception to it. We realise the importance of the teaching of Mathematics, when we find that even highly educated people are not able to do simple calculations. When we find people unable to make use of the knowledge of Mathematics in practical life, we are forced to look at the principles that govern the teaching of the subject. When we find that the knowledge of Mathematics has not been able to help us to calcutate things within the requisite time, we feel that the object or the aim has been defeated. This thing can be rectified only when we have a look at the principles of Mathematics.

Principles and Methods.—Methods of the teaching of Mathematics are also guided by principles. It is the principles that form the basis of the method as well. In fact, the principles are the basic requirements that govern the teaching of the subject. Let us now talk about the various principles that govern the teaching of Mathematics. It is these principles that should be kept in mind by the teachers while they are imparting education to the students.

Correlation of the subject-matter with life.—No subject has an utility in itself. Every subject has a bearing on life. If a particular subject is able to help life, the subject is ¦important. The problems and the difficulties of life are solved through the knowledge of the subject-matter. This is true of Mathematics as well. Mathematics has a direct bearing on the problems of life. We, while solving several problems of life, take help from Mathematics. The teacher of Mathematics should, therefore, try to correlate the subject-matter of Mathematics with the problems of life. Such a correlation would help the students to keep their interest alive in the subject-matter. Then the students shall be able to realise that the knowledge of Mathematics shall be helpful for their future life.

The teacher, in order to give a practical bias to this principle, should try to cite examples and problems from day-to-day life. If the teaching in the class room is divorced from the problems of life, it does not have much of effect and utility in it. If the teaching in the class room is correlated with life, the student shall get to know that only theoretical knowledge is not sufficient ; the theoretical knowledge has to be put into practice.

Knowledge of the importance and need of the subject-matter.—The need and utility of the subject-matter should also be explained to the students. Unless it is done, the students are not interested in the teaching of the subject. The teacher should, therefore, try to explain the importance and the need of the teaching of Mathematics to the students. If the teacher succeeds in doing this, he lays down an important principle before the students. This can be done by correlating the teaching of the subject-matter with individual and social life. This thing is also helpful in creating interest in the students for the subject matter. Once the students are clear about the importance of the subject matter, they are also interested in the subject. Then what is taught to them by the teacher is important for them.

There are no principles to explain the importance of the subject-matter to the students. This can be done by taking into account the psycho-physical requirements of the students and the environment in which they are placed. Given below are the three methods of explaining the importance of the subject-matter :—

By putting some problem of every day life before the students.—The importance of the subject may be explained by putting some problem, drawn from every day life, before the students. This would, on the one hand, explain the importance of the subject-matter, and, on the other, also explain to the students the correlation of the subject-matter with life. Suppose students of a primary class are to be taught addition, they may be asked to bring in certain pieces of wood or pabbles, that are play-things for them. Now every student may be asked to pile up the pieces of wood or stones in the numbers of 5, 7, 10 and 12. Then all these heaps may be mixed together and the students may again be asked to put the heaps in the original form. Then, they would easily learn that if the four heaps containing 5, 7. 10 and 12 pieces of wood or stones are mixed together, they shall come to 34. Then the teacher may explain to them that 5+7+10+12 is equal to 34.

Similarly, if the students are to be taught about multiplication, it may be done by taking a problem of purchasing the sugar from the shop of a provision seller. Suppose the sugar sells at the rate of Rs.1.25 per kg. and the students have to find out the price of 4 Kg. of sugar, then they would have to multiply $1\frac{1}{4} \times 4$. They would work out to 5.00. Then the teacher may tell that $1\frac{1}{4} \times 4$ is equal to 5.

By awakening the natural tendency of the students.— This can be done by correlating the present problem with the interesting thing that has already been studied. It may also be done by exciting the activity of the students. This would make the teaching interesting as well as helpful.

Selection of the subject-matter, keeping in view the intelligence and psychological requirements of the students.— The selection of the subject-matter should be done, keeping in view the psycho-physical requirements of the students. If this is done, this shall warrant activity in the students. This shall also help them to have proper understanding of the real problem.

Awakening the curiosity in the students.—If the subject has to be taught properly, the natural curioisity should be awakened amongst the students. This awakening of the curiosity would create interest in them and would develop their attention towards the subject. This can be safely done for elementary classes by introducing playway method. Later on, according to psycho-physical requirements, other methods such as cross words or Quiz etc. may be resorted to. If by allowing them to resort to these methods, an attempt is made to explain the methods also, the teaching would be interesting and useful.

Keeping the interest alive.—The teacher should always try to keep the interest alive. This can be done by making the teaching practical and useful for day-to-day life. Interest can be awakened by correlating the subject-matter with the problems of life. As far as possible, the problems should be drawn from every day and day-to-day life

POINTS FOR REVISION

(1) Teaching has to be based on certain principles. It is these principles that govern the teaching and help it.

(2) The first principle of the teaching of Mathematics should be to correlate the subject-matter of Mathematics with the problems of life.

(3) The teacher should try to explain the importance of the subject-matter of Mathematics to the students. It may be done by the following two ways:—

(*a*) By putting before them the problems of day-to-day life ; and

(*b*) By correlating the present subject-matter with the subject-matter already taught.

(4) The teacher should try to select the subject-matter of Mathematics, in accordance with the psycho-physical requirements of the educands.

(5) As far as possible, the natural curiosity of the students should be kept alive. It may be done by resorting to methods that are psychologically and physically correct and scientific.

(6) The interest of the students should always be kept alive. This may be done by giving a practical bias to the theoretical knowledge that is being imparted in the class room today.

CHAPTER VI

METHODS OF THE TEACHING OF MATHEMATICS

Q. Describe briefly the various methods of the teaching of Mathematics ?

Or

Describe the Analytical Method of the teaching of Mathematics ? Bring out its merits by giving illustrations ?

Answer. Importance of the methods of teaching.—For teaching of every subject method is very important. It is nothing but a scientific way of the presentation of the subject, keeping in view the psychological and physical requirements of the students. It is through method only that it is possible to make a subject interesting and useful. Without a method, teaching would be pretty hard and difficult. The arrangement of the subject-matter and its presentation is quite important for successful teaching of the future citizens of the country.

Importance of the methods of the teaching of Mathematics.—What is true of other subjects is more true of Mathematics. Generally students are afraid of studying Mathematics. Results of public examinations also indicate that students do not do very well in Mathematics. This is due to several factors, method being one of them. It is very necessary to have a scientific and interesting method of teaching. Without it, would not be possible to improve the teaching of Mathematics and make the students interested in this subject.

Mothods of the teaching of Mathematics shall have to differ from stage to stage and from age to age group. The students of the age group studying in Class VI to VIII cannot be taught by the same method that shall be applied in regard to the students studying at the High school standard.

Methods of the teaching of Mathematics.—As in other subjects, as a result of serious thinking, researches and experiments, following important methods have been evolved for the teaching of mathematiots:—

1. Analytical and Synthetic Method of the teaching of Mathematics.

2. Inductive and Deductive Method of the teaching of Mathematics.

3. Lecture and Heuristic Method of the teaching of Mathematics.

4. Psychological and Dogmatic Method of the teaching of Mathematics.
5. Laboratory Method of the teaching of Mathematics.

Old methods of the teaching of Mathematics.—The methods that we have enumerated above are comparatively of recent origin. They have no doubt elements of the old methods of the teaching of the subject. There was a time when old methods of the teaching Mathematics were supplied. These methods did not determine the psycho-physical requirements of the students. They only concentrated on the subject-matter and tried to make the students learn it through rote memory or cramming. However, educationisits realised the defects of these methods and certain movements for the reform and re-organisation of the methods of teaching were carried out. Ultimately, the methods enumerated above came to the forefront on the basis of scientific and psychological thinking. It shall be better to study those methods a bit in detail.

1. Analytical and Synthetic Method of the teaching of Mathematics.—These are two methods that have characteristics different from each other. In Analytical Method we proceed from unknown to known while in Synthetic Method we proceed from known to unknown. Generally it is believed that the Synthetic Method in which we proceed from known to unknown is easy for the students to grasp. It can be demonstrated through the help of the two examples given below :—

(1) In Synthetic Method we shall proceed as given below :—

A is true and because B and C are equal to A

∴ B is true and ∴ C is true.

(2) In Analytical Method we shall proceed like this :—
C is true if B is true ;

B is true if A is true ;

But A is true ∴ C is true.

It Synthetic Method certain facts are given and on the basis of those facts certain new facts are developed and ultimately the desired result is achieved. On the other hand, in Analytical Method, we start from unknown and ultimately reach the desired result and go on proceeding.

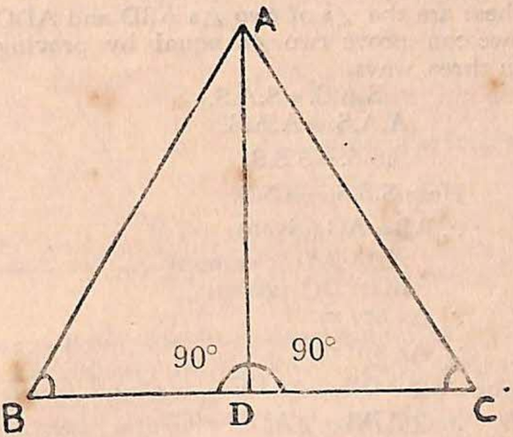
Comparison between Synthetic and Analytic Method.— In the teaching of Mathematics both these methods are used. It shall, therefore, be useful to compare and contrast the two methods.

Synthetic Method	*Analytic Method*
1. In Synthetic Method we proceed from known to unknown.	1. In Analytical Method we proceed from unknown to known.
2. Here certain known truths are put together and then by combining these truths an unknown truth is found out.	2. Here statements are not combined but they are put apart and as a result of this putting apart, the truth of the statements is easily found out.
3. In Synthetic Method we proceed from the data to what we want proved In other words, here we proceed from hypothesis to conclusion. The method is, therefore, thought concise, simple and short. Students here work forward, which is, in other words, the aim and the procedure of education.	3. In Analytical Method we proceed from unknown to known. In other words, we proceed from conclusion to hypothesis and the students have to work backward. This method is, therefore, called tedious, lengthy and time-consuming.
4. In Synthetic Method we reach the conclusion no doubt but no explanation or reason is given for every step taken forward.	4. On the other hand, in Analytical Method every step is called a reason and sequence. In other words, we reach the truth.
5. In Synthetic Method it is difficult to recall what has been forgotten although it is said to be a method meant for learners.	5. Analytical Method is a method for discoverers. It encourages originality. Any step that is forgotten can be discovered easily.

Synthetic Method	*Analytic Method*
6. Through Synthetic Method memory is encouraged and information is gathered. It does not lead to the development of the power of reasoning.	6. Through Analytical Method originality is encouraged and the power of reasoning is developed. There is no scope for memorizing. It encourages the formation of habit.
7. Synthetic Method is not scientific because it has no element of Heuristic Method in it.	7. In Analytical Method there is a lot of element of Heuristic Method and, therefore, it is called a scientific method.
8. Through Synthetic Method it is possible to learn only one thing at a time.	8. Through Analytical Method, while proceeding from unknown to known, the students learn many things. While reaching the solution, they get many things, in the way.
9. Synthetic Method is logical and deductive in character.	9. Analytical Method is psychological and here inductive method is applied.
10. Synthetic Method is formal, certain and more finshied.	10. Analytical Method is informal and tentative.
11. Synthetic Method is a result of thought and in this method teacher is not in touch with the class. In other words, it may be said that it is an end.	11. Analytical Method is a means. It is a process of thinking. Here the teacher continues to be with the class. He carries the class with him and so it is not an end.

Examples of Analytical and Synthetic Methods

The medium to the base of an isosceles triangle is ⊥ to the base.



Given :— An isosceles △ABC, in which side AB = AC.
AD is the median to BC such that side BD = DC.

To prove that :— AD is ⊥ to BC

Synthetic :— Proof :-- In △s ABD and ADC

AB = AC (given)

AD = AD (common)

BD = DC (given)

∴ △s are ≡

Hence ∠ADB = ∠ADC

But these are adjacent supplementary ∠s.

∴ each is equal to 90°.

Hence AD is ⊥ to BC.

Q.E.D.

Analytical Method

To prove that = AD is ⊥ BC
Proof If AD is ⊥ to BC.
∴ ∠ADB = 90°

But ∠ADB can only be equal to 90° if the supplement ∠s are equal.

$\because$ We have to prove
that $\angle ADB = \angle ADC$
which is the supplement of $\angle ADB$.
But these are the $\angle$s of two $\triangle$s ABD and ADC.
Now we can prove two $\angle$s equal by proving these $\triangle$s$\equiv$ $\triangle$s are$\equiv$in three ways.

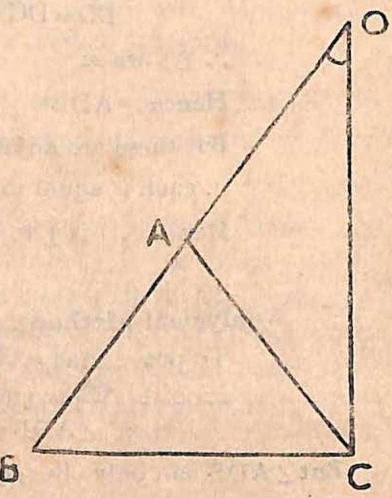
S.A.S. = S.A.S.
A.A.S. = A.A.S.
S.S.S. = S.S.S.
Here S.S.S. = S.S.S.
$\therefore$ AB=AC (given)
AD = AD (common)
BD = DC (given)
$\therefore$ $\triangle$s are $\equiv$
$\because$ $\triangle$s are $\equiv$
$\therefore$ $\angle ADB = \angle ADC$
$\therefore$ $\angle ADB = \angle ADC = 90^{\circ}$
$\therefore$ AD is $\perp$ to BC.

Example : Sum of the external $\angle$s of a $\triangle$ is equal to 4rt $\angle$s. and

Sum of the $\angle$s of a $\triangle$ is equal to 2rt $\angle$s.
Problems :—

Construct a $\triangle$ with base and sum of two sides and a base $\angle$.
Suppose we have to construct a $\triangle$ whose base is equal to $3''$, and some of the side$=7''$. Base $\angle B = 60^{\circ}$.

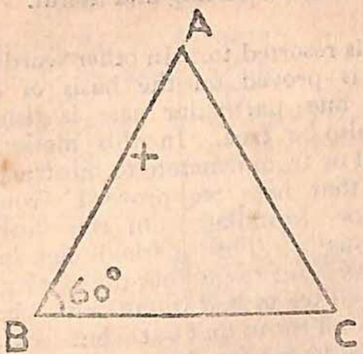
Synthetic Method :—
Take a line $BC = 3''$
Make an $\angle$ equal to 60° at B.
Cut of $BO = 7''$
Join OC
Make $\angle ACO = \angle BOC$.
$\therefore$ ABC is the required $\triangle$.



Analytical :—

First of all we can make a rough diagram.

Here is a $\triangle$ABC
We are given $BC = 3''$
$AB + AC = 7''$
$\angle B = 60^{\circ}$



Thus we can clearly form a triangle of the following order.

Conclusion.—From the foregoing examples it is clear that in synthetic method we do not know why and wherefrom a a step but in the analytic method we reason for each and every step. The analytic proof is lengthy and almost awkward but it is based upon a sound footing and if forgotten, it would be much easier for the student to recall and reconstruct the proof for himself. The synthetic proof is shorter and more elegant and is a special device but it has no sound or general footing. Once forgotten it cannot be reacalled or synthesised. A synthesis shows that every step is true, but does not explain why a particular step was taken. It convinces the reader that the result is true but does not reveal the process adopted why the sequence of arguments was selected. An analytic, on the other hand, is lengthy and not elegant, but it is the only method that explains why and where of each step taken. It is the method by which student can hope to discover proofs, or to recall when they forget.

Really speaking, both these methods are complementary and inter-dependent. In order to make the teaching effective, both these methods should be applied together. The student should be encouraged, through Analytical Method, to provide co-operation to the teacher. Then the teacher should provide, through Synthetic Method, help to the students to learn the subject-matter. If both these methods are applied, the teaching shall be interesting as well as complete. The following words of Arthur Schultze justify this statement :—

> "Analysis is the method of discovery, synthesis is the method of concise and elegant presentation.',

2. **Inductive and Deductive Method of the teaching of Mathematics.**—These two methods are also complementary to

each other. Generally, both these methods are applied in the class-room in order to make the teaching interesting and useful.

In Inductive Method, induction is resorted to. In other words, in this method an universal truth is proved on the basis of a particular truth. It is said that if one particular case is true, the next case of the same type shall also be true. In this method we proceed from particular to general or from concrete to abstract. In other words, it may be said that here we proceed from special example to general rules or formulae. On the basis of certain facts, generalisation is made. This method can be illustrated by an example. Suppose we want to find out the method or formula of simple interest. We shall try to find it out with the help of an example. We shall first of all try to find out what shall be the interest on Rs. 400/-at 6% per annum for four years. We shall come to the conclusion that simple interest for Rs. 400/- at the rate of 6% per annum for four years shall work out to :

$$400 \times 6 \times \frac{4}{100} = 96$$

On the basis of this particular case, the following general rule shall be evolved :—

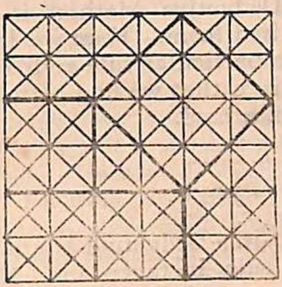
$$\text{Simple interest} = \text{Principal} \times \text{Rate} \times \frac{\text{Time}}{100}$$

Deductive Method is based on deduction. In this method, we proceed from general to particular and from abstract to concrete. In other words, it may be said that the process here is just revers to Inductive Method. It is believed that the Deductive Method is logical and generally the results arrived at by this method are quite correct and unflinching. Here, first of all, a general principle is put forward and then conclusions are drawn. An example will make the thing more clear. Suppose we have to find out the formula about finding out simple interest. In this method, we shall, first of all, say that :—

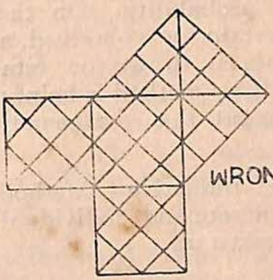
$$\text{Simple interest} = \text{Principal} \times \text{Rate} \times \frac{\text{Time}}{100}$$

After this, we shall continue to apply this formula to all the cases where we have to find out the simple interest. The method shall then be quite simple and easy. For example, interest of Rs. 400/- at the rate of 6% per annum for four years would be $= \frac{400 \times 6 \times 4}{100} = 96$

Example 1 :—Inductive, Pythagoras Theorem



RIGHT WAY



WRONG WAY

Deductive :—In a right angled triangle the square on the hypotenuse is equal to sum of the squares on the sides.

Ex. 6. Find the simple interest on Rs. 1500 at 5% for 6 years.

By Inductive Method :—

S. I. on Rs. 100 for 1 year = Rs. 5

S. I. on Rs. 100 for 6 years = 5×6

S. I. on Re. 1 for 6 years = $\frac{5\times6}{100}$

S. I. on Rs. 1500 for 6 years Rs. $\frac{5\times6\times1500}{100}$

$= \text{Rs. } 450.$

By Dedutctive Method :—We have

$$\text{S.I.} = \frac{P\times R\times N}{100}$$

When P = Principal

R = Rate %P. A.

N = No. of years

Here P = 1500 R = 5% N = 6 years

$\therefore$ S. I. = $\frac{1500\times5\times6}{100}$ = Rs. 450

Let us now examine both these methods in a comparative manner :—

Inductive Method	Dedutctive Method
1. Inductive Method is not absolute and conclusive. It is based on probability. On the basis of certain facts observed, a general rule is drawn or established. This method is helpful for finding out the mathematical facts.	1. Deductive Method is a generalised form. Here we lay down a particular rule and on the basis of that we find certain particular case.
2. In Inductive Method certain concrete and individual cases are taken up.	2. In Deductive Method general rule is taken up. No heed is paid to individual cases.
3. Inductive Method is good for beginners. It creates interest.	3. Deductive Method is not at all meant for beginners. It is a final statement. Here there is no likelihood of exploration but facts are counted.

Place of Deductive and Inductive Method in Class Room.—For class room teaching, both these methods should be applied. If one method is used without having any reference to the other method, the teaching shall not be complete. Beginning should be done through Inductive Method and then an end should be done in Deductive Method. In the teaching of Mathematics this is the correct process. For learning Inductive approach is good but for perfection Deductive approach is best.

In early stages induction predominates. It is the later stage that deduction is complete. In Mathematics discoveries are made through Inductive Method and formal proofs of results are made for Deductive Method.

Inductive Method is psychological.—It creates interest in the student and attracts him towards the problem. It Deductive Method the final stage or the last stage has to be reached. While applying deduction, great care should be exercised so that no wrong creeps in.

It is through deduction that formulae are discovered and found out. In fact, deduction is nothing but a generalised form of the Problem.

In deduction there is a good deal of abstraction which is not suited to the beginners. It should invariably be applied to those who have reached a higher stage of the study of Mathematics.

Deductions is arrived at only through induction. These two methods are, therefore, complementary to each other. That is why it is said that deduction is the house made of induction and without induction deduction is not possible. In the teaching of Mathematics, therefore, both these methods should be applied.

According to the method of teaching or science of teaching or pedagogy, it is the Inductive Method that is given greater importance. Here we proceed from particular to general. However, in the teaching of Mathematics application of only one method, *i. e.*, Inductive Method, shall not be complete. Without Deductive Method, the teaching shall be imperfect.

3. **Lecture and Heuristic Method of the teaching of Mathematics.**—These are the two methods that are sometimes used together and are complementary. However, for the convenience of study, it shall be wise to study these two methods separately.

Lecture Method.—This method has nothing to do with the arrangement of the subject-matter. It is a mode of presentation of the subject-matter. It is not, at all, possible to teach Mathematics through this method. It may be possible to teach social sciences or at places physical sciences through this method but for Mathematics this method shall not be very useful. It shall not help the students to assemble adequately and easily what is given out by the teacher. In this method the student is a passive listener. In Mathematics it is not possible for the student to learn thing by being a passive partner. He has to be an active participant. Unless he adopts this role, he shall not be able to acquire the knowledge thoroughly.

Merits of the Lecture Method.—Some educationists feel that this method has the following merits :—

(*a*) It is easy, concise and even attractive.

(*b*) It is economical as a good deal of subject-matter can be taught in a short time.

(*c*) It can be used for a larger number of students. One teacher can do justice from 50 to 60 students.

(*d*) It is possible to present the subject-matter in a logical sequence.

(*e*) It gives a sense of satisfaction to the students as well as the teacher. It gives them the feeling that they have learnt something.

Demerits and drawbacks.—Some educationists feel that this method has certain advantages and merits while others feel that it has a good deal of drawbacks in it. Given below are the drawbacks about this method pointed out by these educationists :—

(*a*) It takes a lot of time for young teachers to learn the lessons that they have to reproduce before the students. It is, therefore, not time-saving.

(*b*) This method is most unmethodical, because it provides only information which is not at all enough for the students of mathematics.

(*c*) Because the students are passive listeners, so it is possible that they may not rivet their attention on the topic that is being taught. Their attention may wander due to lack of maturity.

(*d*) In Lecture Method ideas follow each other rapidly. It is not easy for students to comprehend the chain of the ideas properly, if their attention is diverted a life.

(*e*) Since it is not easy for the students to comprehend ideas rapidly, if they miss one idea, they may not understand the whole topic at all.

(*f*) This method does not keep the teacher in contact with his students. There is no room for individual attention.

(*g*) The experiment side is totally neglected by the teacher.

(*h*) It is a kind of spoon-feeding and several faculties of the students like observation, independent thinking, etc., seldom get an opportunity to grow.

(*i*) This method is not at all helpful for the teaching of Mathematics, where a good deal of figure work has to be done.

(*j*) Students, due to insufficient preparation, lack of maturity and several psychological drawbacks, do not get very much benefit by this method.

Heuristic Method.—This is like the Lecture Method a mode of presentation of the subject-matter before the class. The word "heuristic" has been derived from the Greek word "heurisco" which means "I find". This method implies that the attitude of the students shall be that of the discoverers and not of passive recipients of knowledge. This method was originally introduced by Armstrong for the learning of sciences. In this method a good deal of importance has been given to experiments. Here the student is made to learn and to think. He is not only supplied with the product of thinking. From this consideration, this is opposed to the Lecture Method where the student is supplied with the product of thinking.

In this method the teacher becomes an onlooker and the child tries to move ahead independently without any help or guidance.

Heuristic Method and the teacher.—The teacher has to adopt the heuristic attitude, whether he teaches the subject through Inductive Method or Deductive Method or Lecture Method or Synthetic Method. This implies that the teacher should try to avoid interfering too much with the work of the students. He should allow the students an independent outlook.

Pre-requisites or requisites of the Heuristic Method.—Heuristic Method cannot be employed without certain pre-requisites. The teacher has to keep in mind certain things before using this method. Following are the pre-requisites of this method :—

(*a*) The teacher must develop the heuristic attitude and make a lot of preparation for it.

(*b*) The question should be so planned that it may be possible for the students to find the solution independently by proceeding in the proper direction. It is required because in this method the pupil is not told anything but he is led to see the solution through carefully planned and well-directed questions.

(*c*) This is not a suitable method for the beginners because in the early stages, students require a good deal of guidance. This guidance should scientifically be planned.

(*d*) This method requires a good deal of planning. Several precautions have to be taken so that things may be planned properly and the lesson may be effective.

(*e*) The teacher and the student should not be discouraged by the slow progress that is made in this method. This method proceeds very slowly and the progress is not very marked.

(*f*) The teacher should realise his responsibility properly. He should know that in Heuristic Method, the students are not to be left alone without any support. Their difficulty should be properly solved and they should be given proper guidance.

Merits of the Heuristic Method.—(1) This method makes the students active participants in the process of learning. It allows them to have independent thinking.

(2) It inculcates in the students the interest for the subject they are studying. It also develops willingness in them.

(3) The teacher is in contact with the whole of the class and there is very little work to be done by the students at home.

Demerits and dangers.—In spite of the fact that this method makes students independent and self-reliant, it has the following demerits and dangers in it :—

(1) This method is very slow and the progress is not rapid. Due to slow progress, students and the teachers are not satisfied.

(2) It is not a good method for the beginners.

(3) When the students do not get help from the teachers, they are likely to consult books, answer books, etc. Such a things are more dangerous.

(4) If this method is not handled properly, it is likely to frustrate the students. Children are not mature and grown-up people and they do not have much of comprehension. While using this method, the psychology of the students has to be kept in mind. Once it is not done, the method falls flat.

(5) It is not possible to expect every student to be a discoverer or a great scientist.

(6) In this method the teacher has to very careful. He has not only to make preparation, but also to be alert throughout the period. Once there is a lapse on the part of the teacher, the whole method is spoiled.

Importance of Heuristic Method.—Heuristic Method is quite useful for teaching Mathematics. It helps the students to discover things independently. It is not possible to apply this method in its full form, but it can be applied in a scientific manner.' The teacher must maintain a heuristic attitude of keeping himself away from the students and allowing them to do things independently. This would help the students to think independently, which is the requirement of these days.

4. Psychological and Dogmatic Method of the teaching of Mathematics.—Dogmatic Method has a bearing on the attitude of the teacher as well as the mode of the presentation of the subject-matter. In this method, teaching of mathematics is conducted with rigour. Students are made to pactise things rigorously. Believers in this method are of the view that once the rigour is given up, the purpose of the entire method is defeated. In this method, the students are asked to memorize formulae and cram the rules and definitions.

Advantages or merits of the Dogmatic Method.—(1) Advantages of this method are of the view that is saves time, energy and does not allow students to get involved in unnecessary thinking.

(2) It gives exercise to memory and helps its growth. They also say that rigour is quite useful for the development of the mathematical aptitude of the students.

Disadvantages or demerits of the Dogmatic Method.—(1) Those persons who do not agree with this method call it unscientific and unpsychological.

(2) This method kills the initiative in the students and makes them a memorizing machine.

(3) This method is unpsychological, because it lays stress on the subject-matter and not on the students and the pupils who are to be taught.

(4) The main object of teaching is to develop certain qualities in the students. The method fails to achieve that object and so it has no use for practical life.

(5) If this method is applied and the students forget the rules, they cannot solve a problem.

(6) The method provides only information and does not allow students to do things independently.

(7) The method makes the teaching uninteresting and dull and converts the whole atmosphere of the class into a mechanical atmosphere.

(8) Through this method, only students with a good memory can make progress. Those students who do not have a very strong memory shall not be able to use this method properly as they would not cram up the rules and memorize the problems.

Importance of this method.—In spite of all the drawbacks, this method cannot be condemned wholly. It has its own utility, but at a proper stage. Given proper circumstances and situations this method can help the teacher to cover a good deal of syllabus in a short time.

Psychological Method.—This is the opposite of the Dogmatic Method. In this method a good deal is laid on the psychology of the students. This aptitudes and faculties are kept in mind while teaching them the subject-matter. This method attaches very little importance to the teaching of the subject-matter and lays real importance on the development of the psychology and faculties of the students. On account of its laying a good deal of stress on psychology, this method is called scientific and useful.

Psychological method makes the teaching of the subject-matter interesting because here the faculties of the students and their psycho-physical requirements are kept in mind.

Precautions.—While employing this method, it is necessary to exercise certain limits and restrictions. This method should not be so employed that the purpose of the teaching is completely lost and only the method becomes the aim. It should be invariably kept in mind that it is a means only and not an end.

5. Laboratory Method of the teaching of Mathematics.—This is a method which is employed in the teaching almost all of the Science subjects. In this method attempt is made to stimulate the activities of the students and to encourage them to make discoveries. It shall be wise if this method is used as a complementary part of the Lecture Method. If the theoretical portion that is given out to the students through Lecture Method is supported by the Laboratory Method, the result is bound to be permanent and more stable. Really speaking, this method has the elements of Lecture Method as well as Heuristic Method in it.

Requisites.—In order to employ this method, in teaching Mathematics it is necessary to have a laboratory. This laboratory has to be equipped with instruments and other aims of teaching. Apparatuses, instruments and aids have to be properly arranged and organised. Without this organisation, it is not possible to employ the method successfully.

Merits or avantages of the Laboratory Method.—This method has the following merits or advantages :—

1 In this method the teaching proceeds from concrete to abstract, which is the scientific method of the teaching.

2. Here the principle of 'learning by doing' is employed. This is a scientific and psychological method, which provides an opportunity to the children to use their instincts and tendencies and so it is most useful for beginners.

3. Interest is the vital factor of this method. Learning by doing awakens the interest of the students for the subject-matter that they are beginners.

4. **Application of Mathematics.**—In this method the rules of athematics are given a practical form. Students get an oppurtunity to apply the mathematical principles in practical and actual life.

5. **Co-relation with other subjects.**—In this method the students get an opportunity to establish co-relation of one subject with other allied subjects. While learning Mathematics through Laboratory Method, the students also come to learn about Physics, Chemistry and such other subjects.

6. **Proper element.**—In this method the students are allowed to work properly and they can proceed in the scientific manner.

7. Here the teacher and the students come in closer contact. It provides an opportunity to the teacher to understand his pupils. Similarly, studens also get an opportunity to know their teacher. This understanding is helpful in learning.

Demerits and disadvantages of Laboratory Method.—In spite of good features this method has certain weak features and demerits as well. They are enumerated below:—

1. It is a complicated method that requires a lot of planning and organisation. If it is not done, the purpose shall be defeated.

2. It is not a teaching method for mathematics unless preparation for it is made, because through experiments, it is not possible to find out the mathemetical factt easily and quickly.

3. Through this method it is not possible to make progress quickly. The results are sometimes discouraging to the students, as well as the teacher.

4. This method has very little of theoretical part in it. Due to this drawback, sometimes it becomes difficult for the students

to know about the reasoning part of the Mathematics, which is the real pleasure-giving aspect of this subject-matter.

5. Development of the power of reasoning is one of the objects of teaching mathematicts. Through this method, it is difficult to develop this particular power.

Utility of this method.—On the basis of the discussion given above, it can be concluded that this method can be used only as a complementary method. It is not an independent method but a complement of the Lecture Method and it should be applied accordingly.

Certain other methods of the teaching of Mathematics.—Apart from the method given above, there are certain other methods as well that are employed in the teaching of Mathematics. They are given below :—

(1) Genetic Method.

(2) Project Method.

(3) Playway Method.

(4) Montessori Method.

(5) Text-book Method.

Genetic Method.—This method is nothing but an improved form of Heuristic Method. In this method attempt is made to retain the spirit of the Heuristic Method and do away with its limitations. Here the whole class is treated as a co-operating group rather than as a separate individual. This method is also a combination of several procedures that include questioning, providing information, explaining, guiding, etc.

Project Method—It is a method which has been defined as an "unit of wholehearted purposeful activity, carried on preferably in its natural setting. In this method attempt is made to present life as it exists. Here certain problems are posed and students are made to solve them. This method is not very much applied in the teaching of Mathematics. It may not be useful as well for teaching the subject.

Playway Method.—It is a method where teaching is carried out in play. While teaching Mathematics, several games and plays are resorted to. · This method may be useful for the teaching of Mathematics, because it would facilitate the learning.

Montessori Method.—Montessori Method is a well-known method in which certain apparatuses are applied for the teaching of the subject-matter. This method has also been applied for the teaching of Mathematics to the students of the younger age group. This method is not useful for the students of the grown-up age.

Text-book Method.—It is a method in which Mathematics is taught with the help of text-books. It is not at all a fit method for the teaching of Mathematics. Here students cannot be asked to read books or learn the doing of the sums. In the teaching of Mathematics text-books serve only as aids for providing standard sums and problems.

POINTS FOR REVISION

(1) Method of teaching is inevitable for every subject. It is actually the way of presenting the subject-matter before the students.

(2) For the teaching of Mathematics, a method is also important.

(3) The method of teaching of Mathematics has to differ from stage to stage. In other words it means that method has to be adjusted, keeping in view the psycho-physical requirements of the students.

(4) There are two kinds of methods of the teaching of Mathematics :—

(*a*) Old Methods ; and

(*b*) New Methods.

Old methods are those that lay stress on cramming and other things.

New methods have been evolved recently.

(5) Following are the new methods of the teaching of Mathematics :—

(*a*) Analytical and Synthetic method of the teaching of Mathematics.

(*b*) Inductive and Deductive method of the teaching of Mathematics.

(*c*) Lecture and Heuristic method of the teaching of Mathematics (*d*) Psychological and Dogmatic method of the teaching of Mathematics.

(*e*) Laboratory method of the teaching of Mathematics.

(6) Apart from the methods enumerated above, the following methods are often employed in the teaching of Mathematics :—

(*a*) Genetic method.

(*b*) Project method.

(*c*) Playway method.

(*d*) Montessori method.

(*e*) Text-book method.

CHAPTER VII

TEACHING OF MATHEMATICS AND AIDS

Q. Describe briefly the various aids that are employed in the teaching of Mathematics ?

Or

What is the importance of the following in the teaching of Mathematics :—

(*a*) **Text-books.**

(*b*) **Note Books.**

(*c*) **Instruments and apparatus.**

(*d*) **Blackboard.**

(*e*) **Models and charts.**

(*f*) **Films.**

Answer. Importance of the teaching aids or material aids.—Today the teaching of Science has developed a lot. In order to keep the teaching interesting and make it effective, certain material aids are applied. These material aids make the teaching effective, simple and interesting.

Mathematics and teaching aids or material aids.—What is true of other subjects is also true of Mathematics. Teaching of Mathematics can be made interesting and effective with the help of the teaching aids. These material aids are more helpful for the teaching of Mathematics, because there a good deal is to be explained to the students. In the following lines, an attempt should be made to talk about the various material aids that are employed in the teaching of Mathematics.

Text books.—Text-books are standardized collection of the subject-matter that has to be taught to the students. They facilitate the teaching of new concepts and skills and maintaining the knowledge already acquired and helping the correlation of the theoretical knowledge with the practical aspect of life. There are three categories of the text-books :—

(*a*) Reference books ;

(*b*) Main text-books ; and

(*c*) Supplementary books.

Utility of the text-books.—Text-books are very useful in practical life. They, on the one hand, help, the teacher to know what he has to teach and, on the other, they also help the students to revise what has been taught to them in the class room.

Things to be considered in text-books.—Teachers have to select the text-books written by certain competent authors. The following things have to be kept in mind in regard to text-books. These things are to be borne in mind, not only by the teachers who select them, but also by the authors who write them.

Useful content-material.—The content-material of the text-books should be useful for the students of the class for which the book is meant. It should contain the subject-matter which is in conformity with the cultural and disciplinary values of the society and also useful for life.

Proper organisation of the subject-matter.—The subject-mtter of the text-books should be properly organised. It should be so organised that it may be possible for the students to revise what they have learnt in the class and then to read something higher in the next class. As far as possible, the text-books should be written on the basis of the spiral method and there should be proper topical gradation as well. There should be ample opportunity for revision and review.

Proper gradation of the exercises.—The exercises, that are given in the text-books, should be properly graded. They should proceed from simple to complex. While writing book, the interest of the students should be kept in mind. Unless it is done, the purpose shall be defeated.

Material and devices for motivation.—The books should be so written that they are able to awaken intellectual curiosity of the students and also to keep their interest alive in the subject-matter. This, to a very great extent, depends upon the treatment of the subject-matter.

Proper arrangement of graphs, charts, diagrams etc.—In Mathematics several concepts have to be explained. These concepts are abstract things and they need to be explained in concrete forms. The explanation in concrete form can be done with the help of these charts, graphs and diagrams. Charts and graphs etc. are the methods of proceeding from concrete to abstract.

Language and technical terms.—The language of the book should be in accordance with the age group of the students. It should be simple and easy to grasp. The technical terms should be uniform and standardized.

Printing and get-up.—The printing and get-up of the text books should be attractive. The binding should be so as to last at least the duration for which they have to be studied by the students.

Competent writer.—The book should generally be written by the authors who have the experience of teaching and know the principles on which the teaching has to be conducted.

Exercise books or work books.—Apart from the text-books, exercise books and work books are also used as material aids. These works contain the exercises and the space for solving those exercises. The main object of these works is to provide proper exercises to the students. The following are the needs and requirements of these books :—

(*a*) The work books should be uniform.

(*b*) They should be so designated that there is space for solving the exercises and also for correcting them.

(*c*) There should be arrangement for developmental material and provision for evaluation.

Utility of the work books or exercise books.—There has a controversy about the use of these exercise books or work books. Some educationists feel that their use leads to the habit of copying in the sudents. On the other hand, others feel that they develop in the students the habit of working independently. However, a via-media has to be found out.

Synthesis of text-books and exercise books.—The best thing, that is suggested today, it is synthesis of the text-books and the exercise books. The teaching of Mathematics should be conducted on the basis of the text-books with occasional and timely use of exercise books.

Blackboard and chart. —Blackboard is an integral part of the Mathematics class room. It is on the blackboard that the sums are solved and the problems explained to the students. It is the chalk that serves as a pen on the blackboard. Without these two aids, the teaching of Mathematics shall be incomplete.

The blackboard should be well polished and smooth. It should be black in colour and fit for writing with a chalk. Now in some of the foreign countries the colour of the blackboard has been made green. It has been done because that colour is useful for eye-sight. Chalk should neither be very soft that it may break quickly nor should it be too hard as to be unfit for writing. There may be coloured chalks as well for drawing figures of Geometry.

Projectors, compasses, setsquares and rulers for blackboard.—In teaching of Geometry, certain figures like triagle, quadrangle, rectangle, have to be drawn. They should be accurate and proper in size. There should be proper arrangement for handling them for use on the blackboard. This can be done by compasses, rulers and setsquares.

Models and charts.—It is not possible for the students to grasp everything which is abstract. In order to clarify and explain the abstract things, something concrete like models have to be presnted to explain those abstract things. These models help the students to grasp the point. Models also create interest in the students.

Models, as far as possible, should be good looking, natural in colour and, as far as possible, aesthetic. They should also be durable. If the models fulfil all these requirements, they sure to be useful.

Charts also help the teacher to explain the point which otherwise would be difficult to explain. For mathematics this is an inevitable material aid. These charts are drawn on paper with the help of colour as well as pencil.

The charts, as far as possible, should be accurate, interesting and good looking. If the charts do not fulfil these requirements, they shall not be useful for the class room teaching. There may be charts and models of certain geometrical figures such as triangles, quadrangles, reclangles.

These charts and models, on the one hand, have a demonstrative value, on the other, they also have a collective value. They are preserved and kept in the Mathematics class room.

Films and other audio-visual aids.—In the teaching of Mathematics films may be useful. Films may be used to explain the concepts and their relationship. Films may also explain the history of the Mathematice and certain other theoretical and practical aspects of Mathematics.

In India, films are not very much used in the teaching of Mathematics. Really speaking, there is dearth of such films in this country, but in foreign countries it is not so. There are a large number of educative films available in those countries, specially in America. Apart from the films, there are other audiovisual aids such as projectors, slides, gramophone, etc. These aids may also be employed for imparting the knowledge of Mathematics in the class room.

Use of the material aids in the teaching of Mathematics.— India is a country with limited resources. It is not expected to have large collection of such material aids. However, it cannot be denied that the material aids make the teaching interesting and lively.

The teacher has to employ these material aids with caution. He should keep in mind the psychophysical requirements of the students. Material aids should be used sparingly, of course, keeping in view the requirements of the subject-matter. Thus we see material aids are very necessary for teaching mathematics.

CHAPTER VIII

MATHEMATICS CLUBS OR ASSOCIATIONS, MATHEMATICS CLASS ROOMS AND MATHEMATICS LIBRARY

Q. Write short notes on the following :—

(1) Mathematics clubs or associations.

(2) Mathematics class rooms.

(3) Mathematics library.

Or

Write a short note on the recreational method of the teaching of Mathematics ?

Answer.—In fact, the teaching of Mathematics should be carried out not only in the class room but it should also be carried out in form of extra-curricular activities. When the studens are out of the class room, they feel themselves free and so, while playing, they learn many things. This thing may be done with the help of the Mathematics library, Mathematics class rooms and such other associations or organisations that provide forum for the extra-curricular activities of Mathematics. Let us now discuss these things a bit in detail.

Mathematics clubs and recreation.—All like minded people and people having common interest, shall organise themselves into groups and associations. Through these agencies, the aims and objects are strengthened and put forward. Students of Mathematics may also be encouraged to organise themselves into Mathematics clubs and Mathematics associations. Under the auspices of these organisations, discussions and lectures, that help create the interest in Mathematics, may be arranged and organised. Certain games that are based on certain concepts of Mathematics and mathematical problems may also be arranged. This would on the one hand, help the students to take interest in Mathematics and, on the other, it would also get them an idea of the practical use or utility of Mathematics.

For the beginners rooms, walls, and such other things can bring the curiosity.

Organisation and working.—The organisation and the working of these organisations should be properly planned. It should be scientifically organised. The students should be given more initiative and direct hand in the working of these bodies. The teacher should act as a friend, philosopher and guide. While doing all these things, an attempt should be made to maintain the semi-official or informal complexion of these bodies. Unless it is done, the purpose shall be defeated. The membership of these organisations and clubs should be voluntary. The membership should also not be very expensive.

Mathematics library.—Library is an important part of school. It provides an opportunity to the students as well as the teachers, to strengthen their knowledge and also to acquire advance knowledge. Mathematics library should contain books of reference that may be used by the teachers. Then, there should be books that should be useful for the students who want to acquire more knowledge about Mathematics ; then there should be magazines and booklets that have articles and news about Mathematics. There should be books that serve as light reading but provide knowledge of the Mathematics. There may be mathematical frictions and story books that have mathematical theme.

There should also be some books that should be helpful for the teachers to know about the methods of teaching of Mathematics.

The library should be well organised. Different categories of the books should be placed at different places. The teacher should have the opportunity to get the books that he needs and similar should be the case with students.

Along with the books, models and charts, etc. may also be placed in the library.

Mathematics class room and museum.—There should be a separate class room for Mathematics. The setting and the arrangement of the Mathematics class room should create an atmosphere for the study and teaching of Mathematics. The students, after entering that room, would find themselves interested in the learning of Mathematics.

The class room should have different charts and models. There should also be charts displaying the practical use of Mathematics in different fields of knowledge, such as Economics, Commerce, Science, etc.

The class room of Mathematics should have proper arrangement for sitting. It is quite natural to have desks, stools, etc. Black-board of the Mathematics class room should be larger than the normal black-board. There should also be arrangement for shifting it, so that the whole problem may be solved in whole stretch. There should also be proper arrangement for keeping projectors, compasses, etc.

There should also be a table for the purpose of administration. Pictures of eminent mathematicians may also be hung on the walls of the rooms. There should be arrangement for the display of the films and other such things.

It shall be advisable to have a class room-cum-museum for Mathematics. Such a thing would help the students to have several things available at one place.

CHAPTER IX

TEACHER OF MATHEMATICS

Or

THE TEACHER AND THE TEACHING OF MATHEMATICS

Q. Describe briefly the qualities of an ideal teacher and explain the importance of those qualities in a teacher of Mathematics ?

Or

What are the chief qualities and characteristics of a teacher of Mathematics ?

Answer.—"A teacher is more like a gardener who tends each plant, examines water and sees that the plant may take its own nourishment. The teacher should be a guide, a helper and a friend. The teacher must study the child, must know the effect of the environment on the child and should know the laws of learning—for which a study of psychology is necessary."

Importance of the teacher.—The teacher is an integral part of the process of education. He is the person who imparts education to the students. He teaches the pupils the subject-matter that is prescribed in the curriculum. His job is a difficult one. He has to look after the students in a class room as well as outside the class room. Apart from the class room activities, he has to look after extra-curricular activities. If the teacher-taught relationship is good, the teaching is effective.

The job of the teacher.—The teacher has to discharge the following functions :—

1. He has to bring about the successful teaching and build up understanding and motivation among the students.

2. He has to study and organise the learning plans of the students and distribute the load for each student in a proper and scientific manner.

3. It is the teacher who is capable of giving due regard to individual differences. Unless he is able to do that, his purpose remains unfulfilled.

4. It is the teacher who can create in the students the interest for the subject through proper appreciation of the achievement of the students. Through personal contact and knowledge, he can create love for the knowledge.

Requirements of the teacher.—The teacher has to have the following requirements, with special reference to the students :—

(*a*) He should know needs, interest and level of mental ability of the students.

(*b*) He should also have a knowledge of the special aptitudes and talents of his students.

(*c*) He should also be aware of the background and the social qualities of his pupils.

Objectives for the teachers.—The teachers have to have the following objectives before them :—

(*a*) Comprehension of basic concept.

(*b*) Proficiency in fundamental skills.

(*c*) Appreciation of significant meanings and items.

(*d*) Development of desirable and helpful attitudes.

(*e*) Efficiency in making sound applications.

(*f*) Confidence in making intelligent and independent interpretations.

Teacher of Mathematics.—What is true of the teachers in general is also true of the teacher of Mathematics. Apart from the special requirements of the teaching of Mathematics, the teacher of Mathematics is also expected to possess the qualities that are present in other teachers.

Qualifications of the teachers.—The teacher of every subject has to possess, in general, the following qualities and qualifications :—

1. Thorough knowledge of the subject.—The teacher has to possess a thorough knowledge of the subject. This is true of every teacher. Thorough knowledge begets confidence in the students and that begets interest. The teacher should not only know the subject-matter theoretically, but he should have the capacity to explain the practical aspects of that theoretical knowledge. The teacher should have done an extensive study and also intensive one. He should keep his knowledge up-to-date.

This is true of the teacher of Mathematics as well. The teacher of Mathematics should have a thorough knowledge of subject-matter.

2. **Interst in the subject.**—The teacher should have interest in the subject that he is teaching or is going to teach. This interest would help him to acquire thorough knowledge and also help the students to create interest in the subject-matter.

3. **Presentable and impressive personality.**—The teacher should possess an impressive personality. Personality has two aspects :—

(*a*) **Internal.**—Internal qualities mean the true knowledge of the subject. A teacher besides having thorough knowledge of the subject should have the aptitude that the students may grasp what he says or does in the class room. He should make the students laugh by his handling the subject. Mathematics is a dry subject hence to make it lively is the job of the teacher.

(*b*) **External.**—External personality means physical features, dress and upkeep. The teacher should keep himself properly dressed and he should have presentable physical features. A teacher suffering from physical defect is bound to create a non-serious atmosphere in the class room. The same is true about the dress. The teacher should be properly dressed. It does not mean that the teacher should be goddily dressed or dressed in a manner that he may look like a film star. Proper dress means that he should be neat and clean and the clothes that he puts on should fit his body. He should not try to show off.

There should be an attitude of sobriety and seriousness in the teacher. This thing helps discipline.

Apart from the external appearance, the teacher should have healthy qualities. His behaviour should serve as an ideal to the students. Unless he is able to do that, he will not be able to discharge his duties successfully.

4. **Knowledge of the child psychology.**—Every teacher needs to have the knowledge of the child psychology. Without this knowledge, he shall not be able to do justice, either to the subject-matter or the students. Through this knowledge, it is possible for him to know the psychophysical requirements of his students and organise his teaching accordingly.

5. Trained and knowing the methods of the teaching.—As far as possible, the teacher should be trained and have knowledge about the methods of teaching. This is more true for the teachers of the higher secondary classes. Training and the knowlege of the methods helps the teachers to carry out their job effectively.

6. Enthusiasm for the subject.—The teacher should be enthusiastic about the subject that he is to teach. This would create enthusiasm and interest in the students which would help the objective of the teaching. This is equally true about the teacher of Mathematics.

7. Truthful and just.—The teacher should be truthful and just. These are the qualities that form the base of several values in life. A teacher, who has these qualities, is very much respected. This respect begets interest in the subject that the teacher is teaching.

8. Dutiful and alive.—The teacher should be dutiful and alive to his job. If he is alive to his duties, the students have respect for him. The teacher also then tries to make proper use of his time and utilises his energy for constructive purposes. This is in the interest of education in general.

9. Practical and social.—The teacher should have a practical attitude and social qualities. These things help him to earn a place in the society and respect amongst his colleagues and students. A person with a practical outlook is able to explain the practical aspect of the subject and also apply it in practical life. Social qualities help the teacher to acquire several friends and respect amongst the students.

10. Ideal conduct.—The conduct of the teacher should be beyond approach. If it is so, the colleagues as well as the pupils have great regard for him.

11. He should not be irritable and nervous.—A teacher, who is irritable and gets excited easily, is likely to cause difficulties for himself as well as for the students. He shall be annoyed by constant queries by the students which is natural for the boys. He shall also get impatient and shall not be able to explain the subject thoroughly.

12. Capacity to inspire confidence in the students.—The teacher of Mathematics should have the capacity to inspire confidence in his students. This can be done only by example and devotion to duty and certain other qualities. If the teacher can inspire confidence in his students, he can very safely carry them along with himself.

13. Qualities of co-operation and sympathy.—The teacher must possess the qualities of sympathy and co-operation. Unless he is sympathetic, he shall not be able to understand the difficulties of his students and if he does not possess the spirit of co-operation, he shall neither seek it from the students nor give it. Teaching is a co-operative effort in which the teacher as well as the students work together. Unless both the partners co-operate, the object cannot be achieved.

14. Capacity to prove things more by action than by words.—Generally, it is said that example is always better than precept. The teacher has to prove things by example and not by words. If he can do that, he is sure to influence his students very much and make them interested in the subject.

15. Full knowledge of the objectives of the teaching of the subject.—The teacher has to possess a clear-cut knowledge of the objectives of the subject that he has to teach. This knowledge helps in carrying out his job properly. This is true about the the teacher of Mathematics as well.

16. Up-to-date knowledge of the subject.—Unless the teacher possesses up-to-date knowledge of the subject, he shall not be able to do justice to his students. Up-to-date knowledge inspires confidence in the teacher to discharge his job successfully.

17. Capacity of analysis and comprehensive description.—The teaching of a subject requires giving minute details as well as a comprehensive description. Without this, it is not possible to explain things to the students. It is all the more necessary for the subjects that have practical aspect in them. The teacher cannot give demonstration of an experiment unless he has the capacity to give minute details as well as comprehensive description. This is true of the teacher of Mathematics as well. He too has possess this capacity and qualification.

18. Originality.—The teacher of Mathematics like the teachers of other subjects, has to possess the quality of originality. Originality helps the teacher to devise ways and means for imparting knowledge effectively and properly. If the teacher is tied down to the conservative outlook, he shall not be able to do justice to the subject. He shall be very hesitant in adopting new methods and employing new researches. This is more required of a teacher of Mathematics. In Mathematics this is more required because every day new researches being are carried out in the subject. The teacher of Mathematics unless he is original, will not be to make his full contribution to the research in the subject.

19. Studiousness.—Unless the teacher is studious, he shall not be able to make his knowledge up-to-date. Studiousness also generates interest in the students for the subject.

20. Presence of mind.—The teacher must possess the presence of mind. Unless he is endowed with this quality, he shall not be able to solve the difficulties that beset his path in the teaching of the subject in the class room.

21. Aesthetic and artistic outlook.—The teacher of Mathematics, like the teachers of other subjects, must possess the aesthetic and artistic outlook. This thing helps the students as well as the teacher. This outlook also helps the teacher to present things in a proper perspective and in an attractive manner.

Teacher of Mathematics and his qualities.—The teacher of Mathematics is expected to possess all the qualities that are enumerated above. Unless he has all these qualities, he shall not be able to do justice either to his students or to the subject. This is what has been envisaged in the folllowing lines :—

> "The first and the greatest factor in creating interest is a sympathetic, well informed, competent and an inspiring teacher. Not all the devices in the world can bear the fruit of continuing and enthusiastic student interest if they are grafted upon the dead stump of instructions in the hands of an incompetent or uninterested teacher. The truly inspiring teacher must first of all be thoroughly grounded in the subject-matter of Mathematics, well beyond the level of any material which he is expected to teach, in order that he may inspire the confidence and respect of his students. He must have a sympathetic understanding of their difficulties and must be always ready and willing to offer proper guidance and stimulation. Finally he must have an *enthusiastic interest* in his subject and in teaching it. He must believe in its values and its contribution to the educational well-being of the students. Enthusiasm is contagious but not a disease.The same enthusiasm backed by sympathetic and enlightened competence is the only real guarantee of the effective meanings of student interest."

POINTS FOR REVISION

(i) In the process of education, the teacher occupies an important place. It is through him that only education can be imparted successfully.

(2) The teacher has to be very clear about the objectives that he has to fulfil. Only then he shall be able to discharge his duties successfully.

(3) The teacher must possess all the qualities that would enable him to discharge his duties effectively. This is true of the teacher of Mathematics as well.

(4) The teacher of Mathematics is expected to possess the following qualities :—

- (*a*) Impressive personality.
- (*b*) Capacity to inspire confidence and affinity with the students.
- (*c*) The teacher must be endowed with the qualities of sympathy and co-operation.
- (*d*) The teacher should be just, truthful, practical and social.
- (*e*) He should have the capacity to undertake theoretical as as well as practical teaching.
- (*f*) He should have moral qualities and should not be irritable and nervous nature.
- (*g*) The teacher of Mathematics, like the teachers of other subjects, must have interest and enthusiasm for the subject.
- (*h*) He should be of studious disposition and possess up-to-date knowledge of the subject.
- (*i*) The teacher of Mathematics should have the capacity to establish the values of life in the students through his personal example.
- (*j*) He should possess the knowledge of child psychology and the methods of teaching.
- (*k*) The teacher should be clear about the objectives of the teaching of the subject. He should be original and possess the capacity to give minute details and comprehensive description.
- (*l*) The teacher must possess presence of mind and artistic and aesthetic outlook.

(5) With these qualities, the teacher shall be able to create interest in the students about the subject that he is teaching. This is true of the teacher of Mathematics as well.

(6) If the teacher possesses the qualities mentioned above, he shall be able to do full justice to his job.

CHAPTER X

EFFICIENCY AND ACCURACY IN THE TEACHING OF MATHEMATICS

Q. Describe briefly the various ways that may be devised for developing accuracy and speed in the students while teaching Mathematics ? (*L. T.*)

Or

'Speed and accuracy must go together in the teaching of Mathematics.' Why ? Discuss this statement.

Answer. Accuracy and speed.—Teaching of Mathematics is the teaching of a subject which is based on accuracy. In Mathematics two plus two shall always be four. This has to be arrived at quickly. In fact, the purpose of Mathematics shall be lost if the accuracy is not arrived at speedily and quickly. The combination of these two qualities can make the students good mathematicians and successful citizens in different walks of life. They can purchase things quickly, find out area easily and save thousands of people in the event of some natural disaster. In banks, if the accuracy is not arrived at with speed, there may be loss of thousands of rupees. It is, therefore, necessary that the students should be trained in accuracy and speed.

Psychological period for the development of accuracy and speed.—Normally 17th or 18th year is the dead line after which it is not possible to develop these two qualities. The period between 10 years and 18 years is the period when these two qualities must be developed. If the students are not trained in figure work and doing addition and substraction speedily at this age, they are likely to develop hatred for it and it shall be hazardous for their future life.

Correlation between accuracy and speed.—Normally it is thought that speed and accuracy cannot go together. That is why the saying goes "Haste makes waste and waste makes want." But this cannot be said about Mathematics. Here speed and accuracy have to go together. If the students cannot be trained in these two qualities by the age of 18 beginning from 10, the ideal opportunity shall be lost.

Psychologically speaking, these two are not contradictory qualities. They are, in fact, complementary qualities. There is need for habit formation. Through habit formation, these two things can go together. As far as possible, the students should be saved from the habit of cramminig. If it is saved, they shall develop efficiency and also develop the habit of doing things accurately and speedily.

Once we have talked about the speed and accuracy and their correlation, we should now try to study ways and means through which these two qualities may be developed. Given below are the ways and means of developing accuracy and exactitude.

1. **By memorizing and habit formation.**—In the teaching of Mathematics, it is not sufficient to learn certain principles and axioms. What is needed is memorization and habit formation. This can be achieved by exercise and drill. It is always advisable that the habit formation should not be done in a mechanical manner. It should invariably be done in a scientific and analytical manner. As far as possible, the basic principles should be explained to the students. Unless it is done in a scientific and systematic manner, it shall lose its utility.

In order to develop accuracy, along with speed, it is advisable that the exercises that are given should be mostly oral. Oral exercises train the swift working of the mind.

2. **Legibility and proper posting of figures.**—One of the basic reasons for committing mistakes is illegible handwriting and improper posting of figures. Students should be trained to write figures correctly, legibly and in good handwriting. Rough work should not be done along with the fair work. It should be done at some other place. If the figures are posted correctly, there is very little likelihood of inaccuracy. In this regard, the teacher of Mathematics should try to keep an eye on the working of the students.

31 **Ability to copy out figures correctly.**—In Mathematics, and especially in Arithmetic and Algebra, students have to copy out figures from some other place. This copying out has to be done cautiously, properly and correctly. If it is not done properly, it is likely to cause incuracy. In this regard, the students should be trained to copy out figures correctly. This training should start at the primary stage of education. The teacher himself should present the form on the blackboard.

The habit of copying should not be encouraged. Copying out of figures is likely to lead to copying out from others. This would cause greater harm but it is essential in Mathematics.

4. **Verifications of results.**—Verification of results leads to accuracy. If a student by verification finds out that the result that he has arrived at is correct, he feels satisfied and develops the habit of verifying the results. Students should invariably be encouraged to verify the results that they arrive at. If it is not done, they are likely to develop the habit of assuming, may be on probability, that the result that they have arrived at is in correct. This habit is dangerous and injurious for the habit of developing accuracy.

In verification, students should be encouraged not only to verify the last result but they should be encouraged to verify all the steps so that the mistake does not creep in unnoticed.

Analysis of the problems.—Students commit mistakes because they do not try to understand the problems and the questions that they have to answer or find answer of. This habit should be discouraged. The students should be encouraged to understand and analyse the problems. If this habit is developed, they likely to develop the habit of accuracy.

Accuracy of statement.—If an arts student is not particular about the statement that he makes, he is likely to develop the habit of inaccuracy. Students should not be allowed to make lose or incorrect statements. They should be trained in exactitude and accuracy. This would help them in Mathematics and the development of accuracy. Such a habit would be helpful in life as well.

Discouraging incorrect calculations and encouraging correct computation.—This principle is based on the psychological theory of reward and punishment. What is done correctly should be rewarded and what is done incorrect should be discouraged. This is paying in Mathematics. If the students who commit computational mistakes are discouraged from committing such mistakes, and the students who do computation work correctly are encouraged the habit of accuracy may be developed in the students. They shall then regard that accuracy is a thing to be acquired, because it is rewarded. This method is specially helpful for the students of the Junior High School and High School classes.

Diagnostic and remedial tests and remedial teaching.—Today educational psychology demands that the causes of errors must be detected. This is called Diagnosis or Diagnostic Testing. Once it has been done, the processing of removal of those causes starts. This is called remedial teaching. This is very useful in the teaching of Mathematics and in developing accuracy and speed. Attempt should be made to diagnose the causes of inaccuracy of the students and remedial teaching should be resorted to in order to remove those causes.

Ways and means for developing speed.—Once it has been accepted that accuracy and speed can go together, attempt should be made to develop speed in the students. Speed can be developed only by developing certain traits and qualities. Given below are certain ways and means for the development of speed :—

1. **Developing of accuracy.**—Inaccuracy is the greatest obstacle in the development of speed. Once accuracy has been developed, it is easy to develop speed. If a person has done something quickly but wrongly, he should not be rewarded. A person who does things correctly and speedily must be rewarded. It is, therefore, helpful to develop accuracy.

2. **Habit formation.**—It is a well known principle of psychology that habit formation helps a person to do a thing speedily and quickly. Habit formation is also responsible for accuracy. Attempt should, therefore, be made to develop the habit of correct figure work in the students.

3. **Time limit for assignment.**—Students should be asked to do things within a limited time. This would help them to do things correctly as well as speedily. If they are given a lot of time, they shall not develop speed. They may develop accuracy.

4. **Use of short cuts.**—Short cuts should be devised and students should be trained in the use of these short cuts. This would help them to acquire speed as well as accuracy.

5. **Leaving of the use of material aids.**—The material aids are, no doubt, an integral part of the teaching. But in the interest of accuracy, efficiency and speed, in the teaching of Mathematics, they should be discouraged and, if possible, not used at all. It may be useful to allow the students to employ the methods of counting on finger tips or certain pieces of wood etc. but, after a stage, it should not be allowed. They should be encouraged to find out the multiples and dividends orally and through mental work but in geometary and certain other branches of mathematics material aids are necessary.

6. **Discouraging the use of the statements and figure work.**—If a lot of language is employed in doing a work and a

good deal of use of figures made, it is likely to retard speed and accuracy. Students are likely to get mixed up and they shall not be able to find out the results accurately and speedily.

7. Psychophysical needs of the students.—Psychophysical needs of the students should be kept in mind while giving work. Individual differences should also be kept in mind. If the same exercises are given to the bright students as well as the dull students, the dull students are sure to commit inaccuracies, and their speed shall be retarded. On the other hand, the bright students shall be held up because of the slow speed of dull students. In other words, it may be said that principles of psychology should invariably be kept in mind while trying to develop accuracy and speed.

POINTS FOR REVISION

(1) In the teaching of Mathematics accuracy and speed are two important requisites.

(2) It is generally thought that accuracy and speed cannot go together but it is a wrong conception.

(3) Accuracy and speed can go together and must be developed together.

(4) Following are the ways and means of developing accuracy :—

(*a*) By memorization and habit formation.

(*b*) By developing ability in the students to copy out figures correctly.

(*c*) By developing the habit of verification of results in the students.

(*d*) Thorough eligibility and proper posting of figures.

(*e*) By developing the habit of understanding and analysing the problems before trying to solve them.

(*f*) By encouraging the students to make correct statements.

(*g*) By encouraging the students who do calculation work correctly and discouraging the inaccurate computational work.

(*h*) By employing diagnostic tests and remedial teaching.

(5) Speed and accuracy, as already stated, have to go together.

(6) Following ways and means may be employed to develop speed :—

(*a*) By developing accuracy.

(*b*) By forming habits.

(*c*) By asking the students to complete assignment within a limited time.

(*d*) By encouraging the students to use short cuts and training them in the use of these methods.

(*e*) By discouraging the students to employ material aids for doing computational work.

(*f*) By avoiding, as far as possible, the use of language and other linguistic work.

(*g*) By employing psychological principles and keeping in view the individual differences.

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CHAPTER XI

ORAL AND WRITTEN WORK IN THE TEACHING OF MATHEMATICS

Q. Describe briefly the importance of the oral and written work in the teaching of Mathematics ?

Answer. Importance of the oral and written work.—Mathematics is a science of figures and written work. Major part of it has to be done in written form, but its application lies in oral work. If the principles of Mathematics can be discussed orally, it shall be helpful. This is more true about Mathematics than other subjects. In the teaching of Mathematics oral work plays an important role. That was the reason why in olden days it was considered to be an exercise and achievement to solve sums orally.

Oral work keeps the mind alert. This also helps the development of the habit of accuracy and speed.

Oral Mathematics.—Sometimes it is thought that oral Mathematics is different from written Mathematics or these are two different things. Fact is not like that. Oral Mathematics and written Mathematics are complementary to each other. If a student is good at oral Mathematics he can do his written Mathematics easily, quickly and conveniently. In fact, these two things should be taught like that. Oral Mathematics saves a lot of time and cut short the process that would have to be done unnecessarily in written Mathematics.

Advantages of the oral Mathematics.—Once it has been established that oral Mathematics and written Mathematics are complementary, it has to be realised that these two should go together. The teaching of one should be associated by the knowledge of the other. Given below are the advantages of the teaching of oral Mathematics :—

1. **Means of introduction.**—With the help of the oral Mathematics, the subject-matter may be introduced. This would help the students to know the subject at the initial stage and would also help the teachers to introduce the first step of Herbert.

2. **Opportunity for revision work.**—Through oral work, it is possible to revise the lesson that has been taught in the class room. It is also possible to test the rules and processes that have been explained to the students.

3. **Development of speed.**—Through this method, it is possible to develop the speed in the pupils. They grow quick witted and are able to solve problems sharply and quickly.

4. **Practical value.**—Mathematics has a practical value in life. This is more true of Arithmetic. Arithmetic, in order to be really useful in practical life, should be handy. It should help the work being done in the shortest possible time. Oral Mathematics or Arithmetic is helpful in this regard. It helps the solution of the problems easily and quickly.

5. **Economy of time.**—Through oral Mathematics, a lot of time is saved. When time is saved, energy is also saved.

6. **Test of written work.**—It can be used as a test of written work. Several principles and formulae of written Mathematics may be tested orally.

7. **Helpful in the learning of written Mathematics.**—Once a student has become well at oral Mathematics, he enters the field of written Mathematics with confidence. He is then sure of succeeding in that field as well.

8. **Develops originality in the students.**—Written Mathematics is helpful in developing the brain and the mental working of the students. Such a thing is helpful for the development of originality.

Disadvantages and demerits of oral Mathematics.—We have just now finished talking about the advantages or merits of oral Mathematics. It should not be forgotten that every subject has some merits as well as demerits. This is true of oral Mathematics as well. Given below are the disadvantages of oral Mathematics :—

1. **Likely to develop hasty attitude in the students.**—Oral Mathematics is helpful for speed but if it is not conducted properly, it is likely to develop a hasty attitude in the students.

2. **Unsystematic and haphazard outlook.**—In written Mathematics the work has to be done systematically step by step. In oral Mathematics no such thing is required. The students are left free to arrive at the results. Such a thing, if not handled

properly, is likely to develop a haphazard and unsystematic outlook in the students.

3. **Likelihood of inaccuracy.**—If oral Mathematics is not conducted properly, it is likely to cause mistakes. These mistakes creep in on account of the speed and haste and lack of any systematic work.

4. **Waste of time and energy.**—If the work of oral Mathematics is not conducted properly, it is likely to cause mistakes which accompany the haste.

Precautions to be observed in regard to oral Mathematics.—After examining the merits and demerits of oral Mathematics, we must also have a look at the precautions that must be observed in the use of this form of Mathematics. Without these precautions, the demerits and disadvantages are likely to have sway :—

1. **Based on the previous knowledge of the students.**—As far as possible, the oral Mathematics should be based on the previous knowledge of the students. Unless it is done, it shall not yield proper results.

2. **Should be employed as a means of revision and test.**—As far as possible, oral Mathematics should be employed only as a means of revision and testing the knowledge imparted.

3. **Proper gradation of the oral work.**—Oral Mathematics should be properly graded. It should start from simple to complex. While doing this gradation, the psycho-physical requirements of the students must be kept in mind.

4. The teacher of Mathematics, while making use of the oral Mathematics, should try *to create an atmosphere conducive for sharp mental working*. It should not be done in an atmosphere which is full of dullness and melancholy.

5. As far as possible this method *should not be employed in fractional and decimal work* and such other work.

6. The *teacher should not reach to any conclusion about the students on the basis of the oral Mathematics*. He should try to give some practice of this form of Mathematics every day in the class.

Questions to be used in oral Mathematics.—The questions that should be used in oral Mathematics should be able to fulfil one or more than one of the following objectives :—

(1) Recall of the past knowledge.

(2) Comparison of the present with some other aspect of knowledge.

(3) Show contract with certain other things.

(4) Explain the cause and effect.

(5) Should be helpful in evaluation of certain things or illustration of certain points.

(6) Classification and definition of figures and other subject-material.

(7) Proof and plan of the subject-matter.

(8) Analysis and synthesis of the subject-matter and problems.

Written Mathematics and its importance.—Most of the Mathematics, as already stated has to be done in the written form. It is different from the oral .Mathematics. The figures and other things are put into black and white. Here most of the things have to be done on the paper. Memory plays only a secondary role.

Importance of written work in Mathematics.—Psychologically speaking, written work has an important place in Mathematics. It is in the written work only that the concentration of attention can be maintained. In oral work, the attention is likely to be diverted by certain thoughts. This is not so much possible in written work.

Purpose of the written work is Mathematics.—Nothing is valuable unless it has a purpose. This is true about written Mathematics as well. It has the following purposes :—

1. The teacher is able to know the amount of the work done by the students.

2. Through written work, it is possible to test the knowledge imparted orally.

3. It is also possible to make the students do things according to proper rules and processes.

4. It is helpful in conducting classes having large number of students.

Advantages and merits of the written work.—Like the oral work, written Mathematics too has some advantages and merits. They are given thus :—

1. Through written work, it is possible to *have clarity of thought and proper reasoning.* It is helpful in mental development as well.

2. Through this method, it is *possible to solve lengthy problems and complicated sums* that are not possible to be solved by oral method.

3. By written Mathematics it is possible *to find out solutions that shall be without fault and flaw.* Written Mathematics diminishes the chances of mistakes.

4. Written work is *more permanent in nature* and so it is possible to judge the achievement of the students.

5. Ultimately, is it the written Mathematics that is used. It is, therefore, *useful for future life.*

6 . It gives an *opportunity to the teacher to carry out the inspection work and see the students while they are busy in written work.*

Precautions in regard to written work.—If the real object is to be achieved, the written work must complete the following precautions. Without the precautions, it is possible that the mistakes may creep in and it may spoil the entire work. The precautions are enumerated below :—

1. **Instructions about the process of the actual work.**—The teacher must give proper instructions to the students in regard to the work, time-limit and other factors.

2. **Written work must keep in mind the mental requirements of the students.**—It should not be beyond the psychological limits of the students.

3. It should be such as to *keep the entire class busy.*

4. The *questions and the sums that are given should be clear and definite and*, as far as possible, have a bearing on the life of the students.

5. It will be helpful if *the written work is examined thoroughly and properly.* As far as possible, the examination should be done in the class room and in the presence of the students. It should be done in such a manner that other students may also get benefit by it.

6. **Verification of the written work.**—As far as possible the written work should be verified to ensure the correctness of

the results. Unless it is done, the purpose of the written work shall be defeated. While doing the verification, the following things should be kept in mind :—

(*a*) No work should be left unexamined or uncorrected. The students should not be given much of the opportunity to correct their mistakes. There should be very little gap between the actual work of the students and the examination by the teacher.

(*b*) The *students should be asked to go through the corrections and re-write them or fair them out.* This would help them to avoid mistakes in future.

(*c*) The teacher should make *frequent use of the blackboard in the correction work.* This would help the students to correct their mistakes and know them fully.

(*d*) *The students should be made habituated of getting their work corrected by the teacher.* The teacher should try to encourage the students to do things correctly and write such remarks that shall encourage even those students who correct their mistakes. This would enable students to correct their mistakes and do things correctly in future.

(*e*) While verifying the written work, the teacher should also have a look at the corrected work. If he does so, the students shall try to do the written work correctly

Role of Mathematics.—

Subjects	Terms.
Interest	Present worth Discount.
time	days of grace
addition	Making of walls
substraction	getting things used up.
Multiplication	adding a number of times
Division	substracting a number of times
Penetration & Combination	Shop arrangements
Centre of gravity	making things rotate
etc.	

POINTS FOR REVISION

(1) Mathematics is a science of figures and so it has to depend a lot on oral as well as written work.

(2) Oral Mathematics occupies an important place in the teaching of Mathematics because it is complementary to written Mathematics.

(2) Following are the advantages of oral Mathematics :—

(*a*) Apart from being complementary to written Mathematics, it is an effective means of introduction.

(*b*) It provides proper opportunities for revision work.

(*c*) It brings about *development of speed* and has a *practical value.*

(*d*) It saves times and is the test of the written work.

(*e*) It is helpful in the learning of written Mathematics and develops originality in the students.

(4) In spite of all these advantages, the oral Mathematics has the following disadvantages :—

(*a*) It is likely to develop attitude of haste in the students and unsystematic and haphazard outlook.

(*d*) Due to oral Mathematics, there is likelihood of inaccuracy, which brings about a lot of waste of time and energy.

(5) Following precautions may be observed in regard to oral Mathematics :—

(*a*) It should be based on previous knowledge of the students.

(*b*) It should be employed as a means of revision and test.

(*c*) There should be proper gradation of the oral work.

(*d*) An atmosphere conducive for short mental work should be created.

(*e*) This should not be employed in fractional work.

(*f*) On the basis of oral Mathematics, conclusions about the students should not be drawn.

(g) The questions that are employed in oral Mathematics should be systematically and scientifically drawn.

(6) Written Mathematics has an important place in the teaching of the subject, which is psychological as well as practical.

(7) Through written work, it is possible for the teacher to know about the actual amount of work done by the students.

(8) Written work of Mathematics has the following advantages and merits :—

(a) It brings about clarity of thought and proper reasoning.

(b) Through this method, it is possible to solve lengthy problems and complicated sums.

(c) To find out correct and flawless solutions.

(d) It has a permanent nature, and is useful for future life.

(e) In this method it is possible for the teachers to carry out the inspections while they are working.

(9) In regard to written Mathematics, the following precautions should be observed :—

(a) Instructions in regard to the time limit and other factors of the work should be given.

(b) The mental requirement of the students should be kept in mind and the entire class should be kept busy.

(c) Question and the sums that are given should be clear and definite, and the written work should be thoroughly examined.

(d) Written work should be vertified and it should not be left unexamined and uncorrected.

(e) Students should be asked to go through the corrections and re-write them.

(f) There should be frequent use of the black-board.

(g) Students should be made habituated of doing the work correctly and the teacher should have a look at the correction work carried out by the students.

CHAPTER XII

DRILL OR EXERCISES IN THE TEACHING OF MATHEMATICS

Q. Describe briefly the importance of drill or exercises in the teaching of Mathematics ?

Or

What is meant by drill or exercises ? What is the importance of this thing in the teaching of Mathematics ?

Answer. Drill or exercises.—Drill or exercises are nothing but the ways to revise the lesson that has already been taught. It is a means to strengthen the knowledge already acquired. Normally, we find that after some explanation, there is a question given. The object of this question is to test the knowledge imparted or get it revised. Actual drill makes the lines straight, curve and in fours.

Importance of drill.—Drill is very important in the teaching of every subject. It is always wise to give certain questions, after explaining certain methods to the students. This would help strengthen the knowledge acquired by the students.

This is, in fact, a part of the examination. Through this method, it is possible to test the knowledge without creating the nervous atmosphere of the examination.

Drill and the teaching of Mathematics.—Drill and exercises have a valuable place in the teaching of Mathematics. Through them, it is possible to test the knowledge and strengthen the knowledge acquired by the students. It also helps the students to revise their lessons.

Advantages and merits of the drill.—Drill has the following advantages and merits :—

1. It keeps the teacher conscious of his responsibilities. It does not allow him to sleep over the matter for the year because he shall have to teach the students in the next class.

2. It also keeps the students awake. They have to do the work regularly, so that they do not fail in the examination.

3. It also helps the school authorities to know about the work that has been done in the class room.

4. It is also helpful in strengthening the knowledge of the students. It gives an opportunity to the teacher and the authority to award proper position to the students in the class.

5. Through this method, it is possible to distinguish between the dull and the bright students. Bright students shall be able to solve the exercises rapidly and with speed.

6. It helps the teacher to know the weaknesses of the students and provide individual attention to them.

Disadvantages and demerits of the drill or exercises.—In spite of the advantages enumerated above, this method has the following drawbacks :—

1. It gives an opportunity to the teacher to sit at ease in the class and ask the students to solve exercises or do the drill.

2. If exercises are not properly given to the students, it is likely to jeopardise the interest of the students in the subject.

3. If not properly handled, it is possible that the students may ask some member of the family to help them in their work. This would make them dependent upon others. The object of teaching of Mathematics *i. e.,* creation of the feeling of self-dependence, shall be done away with.

Importance of drill and exercises in the teaching of Arithmetic.—Although drill and exercises occupy an important place in the teaching of the Mathematics as a subject, yet its importance is more felt and realised in the teaching of Arithmetic. Arithmetic requires more of practice and exercise. The teachers know very well that rewriting of courts and tables makes the knowledge perfect in lower classes. The teacher of Arithmetic should try to give the exercises that are psychological and proper. He should try to give exercises based on the subject or topic already taught. After examining the achievements of the students and correcting their mistakes, he should proceed further. The teacher should try to encourage the students that do exercises accurately, but he should also try not to discourage the students who commit mistakes.

Precaution in regard to drills and exercises.—This method should be employed with proper care and precaution. If the following precautions are not taken, the purpose is likely to be defeated :—

1. These should be a specific period or a time limit for the drill. The students should be expected to complete their assignment within that time limit. In the period of the subject, a few minutes, say five or ten, may be set apart for this thing.

2. The drill should be convenient for the students of the class. It means that it should be in conformity with the psychological requirements and mental attainments of the students.

3. Exercises should be so given that all the students of the class may do them. This means that the exercises should provide practice to all the students of the class. Arrangement should be so made that no student escapes the work.

4. Individual differences should invariably be kept in mind.

5. While giving exercises, the principle of "proceeding from simple to complex" should invariably be followed.

6. The teacher should, as far as possible, correct the exercises. He should find time to do this work in the class room.

POINTS FOR REVISION

(1) Drill and exercises are an integral part of the programme of teaching. This helps the teacher to revise the lesson and strengthen the knowledge imported to the students.

(2) In the teaching of Mathematics drill and exercises are quite important.

(3) Drill and exercises are more important in the teaching of Arithmetic as compared to Algebra and Geometry.

(4) If, on the one hand, drill and exercises have the advantage of strengthening the knowledge and providing opportunity for revision they also provide an opportunity to shirking teachers to ask the students to do exercises. Such teachers sit at ease and let the students do their work.

(5) Drill and exercises should be employed psychologically and properly.

(6) The teacher should correct the exercises as far as possible in the class room.

CHAPTER XIII

EVALUATION OR EXAMINATION IN THE TEACHING OF MATHEMATICS

Q. What are the different types of test or examination? What is their significance in Mathematics ?

Or

Examine critically the utility of the modern test of mathematical ability and of attainment in Mathematics for the teacher of the Secondary Schools ?

Answer.—"For evaluating pupil's work we need definite measures. Accuracy in the measurement of school products should be used to improve the learning of children and the organisation of the school, and to find out those students who need remedial teaching, especially tutoring or transfer to other work. Examinations, as they are functioning, still dominate all levels of education in India."

Importance of examination of evaluation.—In teaching evaluation, which is normally known as examination, is inevitable. It is this method that enables the teachers to know about the achievement of students. Today one of the aims of education is to bring about behavioural changes in the students in accordance with the objectives of the education or teaching. These are the standards that serve as measures of evaluation or achievement.

In spite of the drawbacks and defects that accompany the examination system today, it cannot be denied that the technique of evaluation enables the teacher to make his teaching effective. On the other hand, it inspires the students to work hard and do their work. The guardians and the parents and also the other persons know about the achievements of the students only through the results of the examination. The interest and the aptitude of the students and their weaknesses and differences are known through evaluation. On the basis of the results of the examination and the evaluation, there is need to arrange the guidance programme and indicate remedial teaching.

Importance of examination system or evaluation and Mathematies teaching.—In Mathematic also, evaluation occupies an important place. The achievement of the students and their interest and the aptitude in regard to Mathematics can only be found out through the results of the examination. In fact, in the teaching of Mathematics it is more required.

In India the entire system of education is examination geared. The main purpose of the teaching is to enable the students to pass the examination. It has a sense of respectability attached to it. If a particular person passes a particular examination, he is considered competent and a person who fails to do that is considered incompetent. The teaching of Mathematics has, therefore, also to keep in view the importance of the examination.

Types of test or examination.—Generally the test or the methods of examination are of two types :—

(1) Traditional or essay type examination ; and

(2) New type test or examination.

Traditional type of test or traditional type of examination has only one form and that is essay type of examination.

New types of examination or test.—These tests are called objective tests. They are of the following types :—

(1) Recall type of test.

(2) Alternate response type of test.

(3) Multiple choice type of test.

(4) Matching type of test.

(5) The completion type of test.

(6) Classification type of test.

Defects of traditional type of test or examination.—Today in the field of education, every day the traditional type of test or traditional method of examination is being discarded. New type of tests are gaining importance. There is reason for that. There are certain defects in the essay type of examination on account of which the entire educational system is spoiled. In order to do away with those defects, the new type of tests have been acquired. Before talking about the new type of tests or new examination methods, the defects of the essay type of examination must be found out.

Defects of the eassy type of examination from the point of view of students.—The essay type of examination is less objective. It keeps the students busy and full of nervous tension. It also enables them to have a selective reading. It is quite possible that the students and the teachers may prepare certain guess papers and chance may favour them and they may acquire many things with the help of those important questions.

The present examination system does not spread over the work for the whole year. It confines and concentrates into the limited period of time. Students start preparation a month or two before the actual dates of examination and for the rest of the year they do not study seriously.

The present system of examination is also harmful for the physical development of the students. The students, when the examination is near at hand, keep awake for the whole night. They give up everything and only study. During this period their life is most irregular. It is harmful for the development of physique of the students. In many respects that retards their development.

Defects of the essay type of examination from the point of view of the teacher.—From the point of view of the teacher, also, this examination system has the following defects :—

1. Since the teacher has to get his students passed at the examination, he does not bother about the completion of the course or the teaching of the whole syllabus or curriculum.

2. The teacher directs his whole programme of teaching towards the examination. He forgets the basic principle that he should teach the students. He then starts teaching the subject to the students in such a way that the students may get through the examination.

3. Because of the examination, the teacher is likely to encourage the students to take to cramming, which is not at all a psychological method of study.

4. This has an adverse effect on the morality of the teachers as well. Since the result of the students at the examination is the criterion of the efficiency of the teacher, he makes everything subservient to the examination.

5. The teacher indulges in guess work and so sometimes he passes away the time without doing the serious work of teaching.

Defects from the point of view of the achievement.—Essay type of examination is not the real test of the achievement of the students. Since the answers to the questions in the essay type examination also involve the capacity to express things properly, the knowledge of the subject-matter is sometimes lost in the wilderness.

The marks obtained by the students are not the real test of their knowledge of the subject-matter. Students get marks because they are able to express and present things properly. It does not necessarily go with the knowledge of the subject.

Subjective element of the examiner also matters a lot. If the examiner is not in his proper mood, he is likely to give less marks. On the other hand, if he is happy, he is likely to give more marks.

Mathematics and essay type of examination.—For Mathematics essay type of examination is not a very correct method of evaluation. It is not possible for the students to present many things in Mathematics. He has, therefore, very little scope for writing out things in detail. On account of this, it is assumed that essay type of examination is not meant for Mathematics.

New type of test or examination and Mathematics.—Since essay type of examination has not been found suitable for Mathematics, the new type of tests are getting their place in Mathematics. This system is considered to be more appropriate. It has the following merits :—

1. **Extensive sampling.**—It this type of examination, students have to answer a lot of questions within a limited period of time. The answers that are to be given are quite small. Either one word has to be written or a figure has to be put down. This has an opportunity for testing the achievement of the students in the subject-matter.

It leaves very litle ground for guess work. The students have to cover the entire syllabus or curriculum.

2. **Objectivity in scoring.**—Since the answers of the students are short and exact as well as definite, it is easy to award marks. It is also possible to prepare scoring as key and mark out questions. Since the answers are definite, there is no likelihood of the subjective element of the teacher influencing the marks awarded.

Defects and demerits of the objective type of test.—Although these tests are considered to be quite advantageous, yet the following defects have been pointed out in them :—

1. This type of examination does not provide any opportunity to the students to learn the language and the art of expression which is very important in actual life.

2. In this system of examination, it is easy for a student to copy out the answers from others.

3. There is also scope for guessing the answer and the guess may be correct.

4. If the tests have not been framed properly, they shall not fulfil the objective.

Requirements of the new type of test.—New type of test can be successfully employed, if they fulfil the following requirements :—

1. **Validity.**—The test should be valid. It should fulfil the objective for which it is meant. In other words, it means that it should be free from the defects that are likely to affect the valid results.

2. **Reliability.**—A test shall be called reliable, if it produces the same result even if it is put to constant testing. All forms of alternatives shall yield the same results. If the test is objective, it is bound to be reliable.

3. **Capable of maintaining the interest of the students.**—The test should be such as to capture the interest of the students and maintain it. In other words, it means that the test should be interesting for the students. They should not lose interest in the test.

Different types of objective test.—We have already seen the various types of objective tests. Let us now describe each of them of bit in detail.

1. **Completion type of test or memory recall.**—This type of test has the questions that are either sentenced or a statement in between the words a blank is left within the body of the material. Sometimes the space is left in the margin which has to be filled in the students. Given below are a few examples of this type of test :—

(*a*) The interest at Rs. 1,000 at the rate of 5% per annum for four years is..........

(*b*) The sum of the two sides of a triangle is greater than the..... ....

(*c*) The sum of the three angles of a triangle is equal to ……………..right angles.

Precaution.—In the construction of this type of test, words matter a lot. The statement or the question has to be so carefully worded that the students do not get confused. If they get confused the purpose of such tests is lost.

Recall type of test.—This is nothing but the completion type of test. Here the recall memory is involved and that is why it is known as recall type of test.

Alternate response type of test or true or false test.—In this system certain questions are set for the candidates. These questions are nothing but certain statements. These statements are true as well as false. The students have to choose the correct answer and write 'yes' or 'no', as the case may be.

These questions are not very difficult to construct. The only precaution, that has to be taken, is that the statements that are put before the students should not be very deceptive. Some of the examples are given below :—

(1) $4a^2-9b^2=(2a-3b)\,(2a-3b)$.

(2) The salary for an office assistant for full one month is Rs. 200/-. His salary for one week would be Rs. 70/-.

Note.—The student shall have to indicate that it is not correct before it.

(3) The sum of the two sides of a triangle is equal to the third side………………

Multiple choice type of test.—In this type of test *a* is given which has several possible answers to it. All these answers are given in the space before the question. The student has to mark out the corret answer.

This type of test requires memory as well as correct knowledge and information. There is also the test of the vocabulary. Given below are certain examples :—

Note.—Underline the correct answer.

(1) The point at which the three medians of a triangle meet is called Centroid or Incentre.

(2) A four side figure whose four sides are equal to each other is called a rectangle, a square, a quadrilateral.

Q. The average of 8·5, 9·5, 10·5 and 11·5 is 9, 9·5, 10 and 10·5.

Answer. Matching type of test.—In this type of test, there are two columns. In one column certain things are mentioned and in the other column certain matching things are to be put down. The matching thing is correct answer to the item already mentioned. Given below are certain examples of such tests :—

1	2	3
10×12	Length × Breadth	
$20-4$	120	
$100-50$	50	
Area of a room is equal to	5	

Classification type of test.—In this type of test the questions contain certain groups of words. Every group contains about five words out of which one is the correct answer to the question. The correct word has to be underlined.

In fact, it involves memory, recognition as well as recall. Given below are certain examples :—

1. Underlined the word that does not apply to the group rectangle, square, triangle, quadrilateral, rhombus.

2. $X+3=7$, $X^2=9$, $(X+1)^2=X^2+2X+1$, $X^2-7X+12=O$, $\frac{X}{5}+2=X-10$.

Requirements of the new type of test.—New type of tests have to be carefully constructed. Unless the following precautions are observed and the requirements fulfilled, these tests are likely to lose the utility :—

(1) Careful planning and construction.

2. These tests have to be carefully planned and constructed. Then only it is possible that they shall serve their purpose. For this, the following things are required :—

(*a*) **Determination of the purpose.**—The basic purpose of the test must be decided first.

(*b*) **Selection of the test in accordance with the purpose.**—Once the purpose has been decided, the test has to be found out that would suit that purpose.

(*c*) **Objective questions.**—The questions that are constructed should be objective to the best possible manner.

(*d*) **Ability of the students to be kept in mind.**—The individual difference and the mental ability of the students have also to be kept in mind. Unless it is done, the purpose shall not be served.

(*e*) **Scoring key to be prepared.**—The scoring key, which shall provide answers to the questions, must also be prepared. Unless it it done, the purpose shall not be served. Without this, it shall be difficult to award marks.

(*f*) **Standardization.**—These tests need standardization. Then only it is possible to do away with the element of subjectivity.

(*g*) **Intelligent planning.**—It is not possible for all the teachers to plan these tests. These tests have to be planned and constructed by intelligent teachers as they require intelligent planning.

Other type of tests.—Apart from the tests given above, the following types of test are also employed :—

(*a*) **Intelligence test.**—The basic object of these tests is to ascertain the mental capacity or the psychological capacity of the boys to be taught. These tests require such questions that would give an idea about the mental age of the students.

(*b*) **Recognition tests.**--These tests have the element of the traditional examination as well as new system of examination. In these tests a figure is drawn or a particular question is given and the students have to recognise it. It may be very easily applied to Geometry.

(*c*) **Logical selection test.**—In this test, several names of a particular object are given The correct and the exact name is to be sorted out by the students. This is also a new type of tests.

What is needed ?—In fact, it is true and premature to think of giving up the essay type of test or the traditional type of test. India has yet to achieve that stage. What is needed is the removal of the defects of the traditional type of tests and the introduction of the new type of tests. This would make the examination system scientific. This is the true of Mathematics as well.

POINTS FOR REVISION

(1) Examination is an integral part of education. It enables the teachers, the students and the guardians to know about the attainments.

(2) In India the whole system of education is examination minded.

(3) Examination minded education or examination directed education has certain drawbacks and defects as well. These drawbacks and defects are found in our education.

(4) In spite of all the defects, the examination or the test cannot be ruled out. They have to be adhered to in order to assess the attainment of the students.

(5) Tests are of two types :—

(*a*) Traditional type of test or essay type of test.

(*b*) New type of tests.

(6) In essay type of tests the student has to put forward what he has learnt.

(7) The essay type of test is getting out of date. Educationists have pointed out that it has defects and drawbacks from the point of view of the students, the teacher as well as the examination results.

(8) The element of subjectivity influences the examination results in the essay type of examination, which is quite dangerous.

(9) In order to do away with the defects of the essay type of tests, new type of tests have been evolved. They are of the following types :—

(*a*) Recall type of test.

(*b*) Alternate response type of test.

(*c*) Multiple choice type of test.

(*d*) Matching type of test.

(*e*) The completion type of test.

(*f*) Classification type of test.

(10) New type of test have to be carefully planned and standardized. It is not possible for all and sundry to construct these tests.

(11) New type of tests have certain advantages as well as disadvantages.

(12) What is needed to day is the combined use of the traditional type of examination as well as new type of tests. Such a thing would ultimately lead to wholesale use of the new type of tests. In India it is not possible to do away with the essay type of examination completely, at least for the present moment.

CHAPTER XIV

CORRECTION WORK IN THE TEACHING OF MATHEMATICS

Or

CORRECTION AND THE TEACHING OF MATHEMATICS

Q. Describe briefly the importance of the 'correction of work' in the teaching of Mathematicts ?

Or

Why is it necessary to correct the written work ? Describe briefly the different methods by employing which the teacher may do the correction-work in Mathematics, with the minimum possible of time and energy ?

Or

What precautions are needed in the correction of the written work in the teaching of Mathematics ?

Answer. Importance of the correction work.—Correction work is an important aspect of the teaching of a subject. If the work of the students is not corrected, the mistakes continue to be repeated and so the purpose of teaching of a subject is more or less defeated. The real object of the teaching of a subject is to correct the mistakes and to avoid their repetition. This can be done only by correction work

Correction of work and the teaching of Mathematics.—In the teaching of mathematics the correction of work occupies a very important place. The work shall not be complete unless the teacher goes through the work done by the students and correct their mistakes. If it is not done, the progress cannot be made. It is, therefore, necessary to do this work effectively and accurately.

Different methods of the correction of work.—In the correction of the work, the teacher has to apply different methods. These methods depend upon the circumstances, *i. e.*, time and conditions. The mental growth of the students is also a determining factor in this regard. It is always advisable that the methods, that are employed, should make the students know about their mistakes. This would enable them to correct those mistakes properly. Normally, the following methods are employed in the correction of the answer books of Mathematics :—

Correction of the work by going around the class.—While the students are busy in the work, the teacher moves about in the class room and goes to each student and corrects his or her mistakes. While doing so, he points out the mistakes and makes out necessary corrections. This process enables the teacher to do the correction work while the students are busy in their work.

Precaution.—This method requires one very pertinent precaution. The teacher should know when he should interfere in the work of the student. Unless it is done, it is likely to have an adverse effect on the progress of the student.

Merits of this method of correction.—This method has the following advantages :—

1. It saves time and energy of the teacher.

2. It enables the students to know their mistakes and correct them in the presence of the teacher.

3. The work of the class goes on while the correction work is also in vague.

Disadvantages of this method.—This method, in spite of the advantages enumerated above, has the following disadvantages also :—

1. It is not possible for the teacher to go around the whole class in one day and every day and do the correction work.

2. It requires the division of the whole class into certain groups and the correction work to be done in groups on certain fixed days. It is likely that due to holidays certain groups may not get their work corrected at all.

3. Unless the correction of the work of the whole class is done, it is not possible to move further and in this method it may not be possible to do it very effectively.

4. It does not save much of time and energy. Instead of the energy of the students the energy of the teacher is spent.

5. It is also possible that while a teacher is doing the correction work, other students may leave their work and start talking.

Correction work on the black-board.—It is quite possible that the teacher may solve the sums on the black-board or he may ask some student to solve them on the black-board. With the help of the correct solution on the black-board, the students may correct their mistakes.

Precautions.--This method should be employed only when the written work has been done. It cannot go along with the written work by the students. Unless this method is evolved, it is possible that the work may get disturbed and neither the correction work can be done nor the written work.

As far as possible, the students should not be asked to correct their mistakes by themselves. They should be asked to exchange their answer books. This would minimize the chances of dishonesty.

Advantages of this method.—This method of correction has the following advantages :—

1. It develops the moral quality in the students.

2. It enables them to correct their mistakes properly and also face the class while they are doing the correction work on the black-board.

3. It enables the teacher to know the students who are able to do the work correctly.

Disadvantages of this method.—In spite of the advantages enumerated above, this method has the following disadvantages :—

1. It is possible that the students may not correct their work properly and they may show as if they have done it correctly. Such a thing has dual disadvantage. On the one hand, it enables them to neglect their work and, on the other, it is likely to develop a dishonest attitude in them.

2. It does not provide for the requisite individual to the students. The teacher does not have to attend to the need of every student. He knows only a few students. He does lnot know the weaknesses of each and every student.

Correction of the work by giving out the correct answer.—In this method, the teacher gives out the correct answer and the students correct their mistakes. It is possible to use this method in mental test also.

Precautions.—While using this method, it should be kept in mind that this method is not useful for the students of lower classes. This method is useful for higher classes.

As far as possible, coloured pencils should be used and the students should not be allowed to correct their mistakes. Exchange of answer books should be done.

Advantages —This method has the following advantages :—

1. It enables the students to know their own mistakes.

2. It helps them to develop a moral attitude and the attitude of self-dependence.

3. In this method, the teacher does not have to spend much of time and energy.

Disadvantages of the method.—Like the advantages enumerated above, this method also has some disadvantages :—

1. If proper precautions are not taken, the students are likely to develop a dishonest attitude.

2. Although in this method the students know their own weaknesses, yet the teacher does not know them. This is not at all a very healthy situation.

Utility.—In spite of all the advantages and disadvantages, this method, like the other two methods, has its utility. It is, as already stated above, quite useful for the students of higher classes.

Correction work with the help of the bright students.—In this method, the teacher takes the help of the bright students. He goes through their work and once he has corrected their work, he asks them to correct the mistakes of their other fellow students.

Precautions. – In this method, the teacher should invariably put up the correct answer before these students as well. The students should be told that what they are doing does not entitle them to treat their other class fellows as their inferiors.

Merits.—This method has the following advantages and merits :—

1. It saves time and energy and trains the bright students in jobs of responsibility.

2. It enables the weaker students to seek help from their fellow students.

Disadvantages.—In spite of the advantages enumerated above, this method has the following disadvantages :—

1. If not properly utilised, this method is likely to develop an inferiority complex in the weaker students. Bright students are likely to grow bully.

2. This method is likely to develop a notion of false self-confidence in the bright students, which may ultimately can harm to the weaker students. Bright students may not correct themselves under the impression that they are very wise and they know everything.

Correction of a few answer books by the teacher and the correction of the rest by the students themselves.—The teacher may take away half a dozen of answer books with him, outside the class room, correct them and bring them back to the class room next day. On the basis of the correction that he has made in those answer books he may ask other students to correct their answer books.

Advantages.—This method has the following advantages :—

1. It saves the time and the energy of the teacher.

2. It enables the students to correct their mistakes by themselves and know them properly.

Disadvantages.—This method has to be used with proper precautions, without which it is likely to cause the following disadvantages :—

1. The students may not do work at all and when they get the corrected answer books, they may copy out the sums.

2. It does not let the teacher have contact with the whole of the class and so he is not able to know the weaknesses of individual students.

3. This method enables the negligent students to have their own way.

Correction of all the answer books by the teacher outside the class.—The teacher may take out all the answer books and correct them and return them to the students. Here the teacher has the opportunity to point out the individual mistakes. He may write notes and provide guidance for future.

Advantages.—This method has the following advantages :—

1. It enables the students to know about the mistakes. It enables the teacher also to know about the mistakes of his pupils.

2. It provides an opportunity to the teacher to plan his lesson in accordance with the attainment of his students.

Disadvantages.—In spite of the advantages, this method has the following disadvantages :—

It is a time consuming affair in which the teacher has to devote a lot of time and spend a good deal of energy. Unless he does so, it is not possible for the teacher to correct the answer books overnight and so the work is likely to be dislocated.

Awarding marks while correcting the work.—The teacher may award marks for the work done by the students, on the basis of correctness of the work. This method enables the teacher to inspire his pupils.

Precautions.—Marking, if done on all the pieces of the written work, is likely to degenerate into a routine affair. Then it shall lose its importance. Marking should, therefore, be done sparingly. The best thing would be to hold weekly, fortnightly or monthly days and award marks. This would also inspire the students to work hard.

This method is useful for lower classes only. It is not a very successful method for higher classes. At this stage, the students come to know about the significance of awarding of marks. This method has, therefore, a limited utility.

Correction and the mistakes.—Correction is done only of the mistakes. The main object of the correction work is to let the students know about the mistakes and help them avoid their repetition. The teacher should know about the causes of the mistakes. Then only it shall be possible for him to help the students in correcting them. It is here that remedial teaching has its significance.

Causes of the mistakes.—Mistakes are not committed without causes. These causes have to be explained and found out. The teacher should try to ward off these causes. These causes can be removed only if they are properly found out. Generally, they are as follows :—

1. **Lack of concentration.**—If the students do not give proper attention to what is being explained to them in the class room, they are likely to commit mistakes.

2. **Improper presentation of the subject material.**—If the presentation of the subject material is not proper, the students are likely to lack concentration. This would not be helpful for them.

3. **Lack of proper drill and exercises.**—Unless the students are given proper exercises and drill, they shall not be able to learn things properly and in that cases they shall commit mistakes.

4. **Defective knowledge of the steps.**—This is, in fact, a defect of presentation. In Mathematics certain steps have to be explained in order to help the students to do things correctly. If one step is not clear to the students, they shall not be able to move ahead and then they shall commit mistakes.

Types of mistakes.—Generally mistakes are of the following two types :—

1. Mistakes of principles.
2. Mistakes of computation.

Mistakes of principles are those that are in regard to theory on the basis of which the work is actually done. If the principles are not clear, the work is likely to be completely defective.

Computational mistakes are the mistakes of working on the figures.

Kinds of computational mistakes.—Computational mistakes are also of the following two types :—

1. Accidental.
2. Consistent.

Accidental mistakes are those which occur by chance or accident. They are not a normal part of the working. On the other hand, consistent mistakes are those that are consistently and continually committed by the students. There is no need to explain the causes of the accidental mistakes, because they occur only by chance and can be corrected.

Consistent mistakes need greater attention.

Correction of the work and clarification of the mistakes.—No correction shall be complete unless the mistakes are properly explained to the students. If the students know about the causes of the mistakes, they shall be able to correct them in future.

There is greater need to correct and explain the mistakes of principles. It is the principles that govern the working and so mistakes that are based on principles are more serious and important.

In regard to computational mistakes, there is not much need of an explanation. These mistakes can be corrected. There is no definite cause. Sometimes there is some psychological cause. Such a cause does not need explanation only. It needs more than that. It has to be tackled on different plain.

Without explanation of the mistakes, correction work is not complete. If the clarification has been properly done, it is possible to avoid the repetition of the mistakes.

POINTS FOR REVISION

(1) Correction of the work is an integral part of teaching. It enables the teacher to know about the weaknesses of the students and provides an opportunity to the students to do away with the mistakes.

(2) In the teaching of Mathematics correction of work is quite important. Without the correction of the work, the teaching of Mathematics cannot be properly done.

(3) In the correction of the work in Mathematics, the following methods are generally applied :—

(*a*) The teacher may correct the work by going round the class.

(*b*) The teacher may correct the mistakes of the students with the help of the black-board.

(*c*) The teacher may deliver the correct answer to the students and correct their mistakes.

(*d*) The correction of the work may be done with the help of some bright students of the class.

(*e*) The teacher may take out a few answer books and correct them and ask other students to correct their answer books with the help of those corrected answer books.

(*f*) The teacher may correct all the answer books of the class.

(4) No method can be said to be complete and perfect. What is needed is the combined use of all these methods.

(5) Marking may also be done while correcting the work of the students. This should be done sparingly.

(6) Causes of mistakes should be found out. Generally following are the causes :—

(*a*) Lack of concentration.

(*b*) Defective presentation.

(*c*) Lack of drill and exercises.

(*d*) Improper knowledge of all the steps.

(7) Following are the types of mistakes :—

(*a*) Mistakes of principles.

(*b*) Computational mistakes.

(8) Computational mistakes are also of the following two types :—

(*a*) Accident.

(*b*) Consistent.

(9) It is necessary to explain the causes and other factors of the mistakes. Without clarification, it shall not be possible for the students to correct their mistakes.

(10) Clarification of mistakes is an integral part of the correction work by a teacher.

CHAPTER XV

PRECAUTIONS IN THE TEACHING OF MATHEMATICS

Q. Describe briefly the precautions that should be taken in the teaching of Mathematics ?

Or

What are the different precautions that ought to be taken in the successful teaching of the Mathematics ? Discuss briefly.

Answer. Importance of precautions and the teaching of Mathematics :—

Precautions.—In the teaching of Mathematics, precautions play an important role. In fact, Mathematics requires a lot of correct process and working. It is here that the element of precaution has its importance. Given below are certain precautions that have to be observed in the teaching of Mathematics :—

1. Proper gradation.—The subject matter of Mathematics should be properly graded. Unless it is done, it shall not be possible for the students to grasp the subject matter. It is a well known principle of teaching that the teacher should proceed from simple to complex. The teacher of Mathematics should also keep this thing in mind. He should take up the difficult items only when simple items have been properly explained to the students and understood by them.

Certain aspects of Mathematics have bearing on other aspects as well. These things should be so taught that what is primary should be put forward first and what is secondary or subservient should be put forward afterwards. In other words, it may be said that decimal should be taken up only when the students have understood the fraction and other allied matters.

2. Short cuts and aids.—In the teaching of Mathematics, such aids and helping devices that are helpful for the teaching of the subject should be properly taken up and utilised. Generally it is not possible for all the students to understand things without taking help from other helping devices. If the teacher takes the precaution of making use of these things, he shall be able to make the students understand things properly.

3. **Language of the process of Mathematics or the language of the rough work or the language of the procedure.**—The working of the sums requires use of some language. This language should be crisp, clear and easy to understand. All the marks of punctuations should be properly used. Without this, it is not possible for the students to understand things properly.

4. **Practice and habit-formation.**—Certain subjects are such that the students have to be imparted knowledge, while in other subjects there is need for giving practice and habit formation. Mathematics is a subject where habit formation is quite helpful in efficiency. This habit formation can be achieved through practice. It saves a lot of wastage and brings about economy in time and energy.

The teacher of Mathematics should take this precaution very carefully. He should try to provide proper practice to the students and help them in their habit formation. If it is done, they are able to do the sums properly.

5. **Practice of oral and written Mathematics.**—The teacher of Mathematics should give practice of written as well as oral Mathematics to his students. The knowledge acquired through oral Mathematics should be tested by written Mathematics and vice-versa.

6. **Knowledge of different steps of practice.**—Mathematics can be solved successfully only if the students are well aware of the different steps in addition, substraction, multiplication etc. Without the knowledge of all these steps, it is not possible to give proper practice. This can be true about Arithmetic and algebra. So far as geometry is concerned General enunciation, Particular enunciation, Construction proof etc. are adopted.

Practice should be given after explaining the theory and the underlying principles. Students should be given practice only when they have been given the knowledge of the theory or the principle. This knowlege of theory helps efficiency in the practice. The teacher of Mathematics should bear this thing in mind.

7. **Improtance of revision**—In Mathematics revision occupies an important place. It is this revision that helps the strengthening of the knowledge and the experience. It is, therefore, advisable for the teacher of Mathematics to make the students

revise what they have been taught. This practice and revision should be regular and timely.

8. **Speed and accuracy.**—We have already talked about it in detail. In Mathematics it is necessary to make the students acquire speed and accuracy. These two things must go together. The teacher of Mathematics should try to develop these things in the students.

POINTS FOR REVISION

(1) Precautions play an important part with doing of things correctly. Precaution is equally important in the teaching of Mathematics.

(2) The teacher of Mathematics should try to observe the following precautions :—

(*a*) The subject-matter should be properly graded.

(*b*) Devices and aids should be scientifically used.

(*c*) The language of the Mathematics should be clear, simple and crisp.

(*d*) Attempt should be made to form habit in the students through practice.

(*e*) Verification of the knowledge of Mathematics should be done by employing written as well as oral exercises.

(*f*) The students should have the proper knowledge of the steps of the work.

(*g*) Theoretical knowledge must precede practice.

(*h*) Revision should occupy an important place in the teaching of the subject.

(*i*) The teacher should try to develop speed and accuracy in the students.

CHAPTER XVI

TEACHING OF MATHEMATICS AT DIFFERENT STAGES OF EDUCATION

Q. Describe briefly different methods of teaching of Mathematics at different stages of education ?

Or

Is it possible to teach Mathematics in the same way at different stages of education ?

Answer. Teaching at different stages of education.—Teaching of every subject differs from stage to stage. This difference lies in aims, objects, objectives, syllabus and methods of teaching. In other words, it means that the teaching of a subject shall be different at the primary stage of education as compared to secondary stage of education. This is true of Mathematics as well. Mathematics differs from one stage to the other. In other words, it means that the teaching of Mathematics at the primary stage of education shall be different from the secondary stage of education.

Different stages of education and the teaching of Mathematics.—Normally, the whole system of education may be divided into the following stages :—

1. Pre-primary stage of education.
2. Primary stage of education.
3. Junior High School or Middle School stage of education.
4. Secondary stage of education.
5. Higher education.

Here we are concerned with the *primary, junior-high school* and *higher secondary* stages of education. Method of teaching is not rigidly applied to pre-primary and higher stages of education. These two stages of education have their own characteristics and qualities. These characteristics and qualities have their own novelties and so the teaching of Mathematics has to be guided in accordance with those special features. It shall not be possible to talk about those feature while talking about the method of teaching of Mathematics. Here we shall try to discuss the teaching of Mathematics at different stages of education.

Primary stage of education and the teaching of Mathematics.—This stage of education starts from class I and goes up to class V. Here the children are still of a tender age. The teaching of Mathematics as this stage of education has to be guided by psychophysical requirements of the students. The curriculum has also to be planned accordingly.

Aims of the teaching of Mathematics at the primary stage of education.—At this stage of education, the main purpose is to teach elementary mathematical concepts, principles etc.

It is also aimed at developing certain habits and skills in the students to enable them to apply the elementary knowledge of the Mathematics in higher stages.

The students have to be taught about numbers and their mutual relationship. Importance of fractions and such other things have also to be taught at this stage of education.

The students have to be made aware of the concept of number and its utility in life.

Syllabus.—The syllabus of this stage of education is expected to contain :—

(*a*) Number, counting etc.

(*b*) Substraction, addition and multiplication tables up to 20.

(*c*) Weights and measures.

(*d*) Decimal fractions etc.

(*e*) Capacities, coins and buying and selling.

(*f*) Knowledge of different parts of measures, such as millimetre, centimetre, metre, etc.

Method of teaching.—At this stage of education, the teacher should attempt at the formation of mathematical concepts and lay the foundation of skills. He should try to give practical work to the students. Learning experiences should be developed and strengthened.

Activities.—All such activities that are helpful in strengthening the learning experiences and the concepts on the basis of the syllabus should be encouraged. They may be like counting in groups, measuring in length, counting of things of the class, collection of things etc.

Teaching aids.—At this stage of education, the children are fond of pictures, coloured charts and such other playable things. All such things should be employed in the teaching of the subject.

Teaching of Mathematics at Junior High School stage of education.—At this stage of education, the students have grown up. They are more realistic as compared to the students of the primary stage of education. Their teaching should, therefore, be more realistic.

Aims of the teaching of Mathematics at the Junior High School stage of education.—Following should be the aims of the teaching of Mathematics at this stage of education :—

(*a*) Development of the mathematical skills.

(*b*) Realisation of the importance of Mathematics in day-to-day life.

(*c*) The teacher should try to develop in the students the computational skill, power of analysis and abiltiy to work accurately.

Syllabus. At this stage of education, the syllabus of education should contain the following :—

(*a*) Numbers and their notation up to crores.

(*b*) H. C. F., L. C. M., Squares, Squareroot, Multiples, and their factors, decimals and their fractions, average, ratio and proportion, percentage.

(*c*) Weights and measures and their different varieties and quantities.

(*d*) Simple interest.

(*e*) Area of different figures.

(*f*) Volume and area of cricle.

(*g*) Parallel lines, construction of triangles, division of lines, angles at the joint of walls. Ceilings, floors, doors, and windows and their importance.

(*h*) Use of letters for numbers and other aspects of Algebra.

Method of teaching.—The teaching of Mathematics as Junior High School stage of education should be more realistic. The students should be given the practical knowledge of the subject and also the practical utility of different subjects included in the curriculum. The teacher should try to develop various skills in the students by givinig them practice.

Teaching of Matehmatics at the higher secondary stage of education.—This stage of education has adolescents and post-adolescents to teach. Students come to this stage of education while they are entering adolescence and leave it when they have more or less completed it. This stage of education includes classes IX, X, XI and XII.

The students of this stage of education have more or less developed a realistic outlook. On the other hand, they also live in the world of imagination. All these things have to be borne in mind while teaching Mathematics at this stage of education.

Aims and objects.—Apart from the aims and objects of the teaching of Mathematics enumerated under Junior High School stage of education, the teaching of Mathematics at this stage should aim at following :—

1. (*a*) "To enable the child to understand and appreciate the part Mathematics plays in his environment and his community such as construction of arches and the hanging walls.
   (*b*) "To enable the child to understand the part Mathematics plays in controlling environment for human progress and in building and running social institutions." *viz.* why man walks straight, various bones of the body are at different angles.
   (*c*) construction of graphs.
2. The teaching of Mathematics at this stage of education should also enable the students to have the idea about the demand of Mathematics in every day life.

Objectives and concepts.—Apart from developing further, the concepts and the objects already enumerated under Junior High School stage of education, attempt should be made to achieve the following :—

1. The students should be made to develop the power to understand and make use of the mathematical language or the symbols used in Mathematics.

2. They should also be made to develop the power of interpretation and expression of statistical and graphical data.

3. The students should be encouraged to develop quantitative thinking and the power of generalisation.

4. The concept of proportion, average, continuity, space relationship, motion, vectors, etc. should also be developed.

5. The concept of arranging shop things in the form of permutation and combination is to be developed. Use of match box cover to teach solid geometry is also to be encouraged.

Syllabus.—The syllabus of Mathematics at this stage of education should include :—

(*a*) Arithmetic.

(*b*) Algebra.

(*c*) Geometry.

(*d*) Mensuration.

(*e*) Coordinate Geometry.

(*f*) Dynamics.

(*g*) Statics, etc.

The construction of the curriculum should be guided by two considerations :—

(*a*) It should prepare the students for higher education ; and

(*b*) It should enable them to enter life.

In other words it means that the curriculum should be complementary to the curriculum of higher education and it should also be helpful for life.

Teaching.—The teaching at this stage of education should be more developed. Attempt should be made to strengthen the experiences already enumerated under Junior High School stage of education.

In regard to the teaching aids, they should not be used in abundance. Graphs and other things, that are helpful in the development of the concepts, should be used sparingly.

Different types of activities, that have a practical value and are also helpful in the teaching of the subject should be encouraged.

POINTS FOR REVISION

(1) Teaching of Mathematics, like the teaching of other subjects, differs from stage to stage.

(2) The teaching of Mathematics should differ from primary to junior high school and from junior high school to higher secondary stage of education.

(3) Teaching of Mathematics at the primary stage of education should be guided by psycho-physical requirements of the pupils.

(4) At the primary stage of education, only elementary Mathematics should be taught. Charts, pictures, etc. may also be used.

(5) At the junior high school stage of education the teaching of Mathematics should be done in a more realistic manner. Here those concepts should be developed that are helpful in practical life.

(6) The curriculum or the syllabus should include Algebra and Elementary Geometry along with Mathematics.

(7) At the higher secondary or secondary stage of education, Mathematics should be taught in more serious and useful manner. The teaching should have a link with the realities of life.

(8) Along with developing further the concepts and objectives enumerated at the junior high school stage of education, the teaching o Mathematics at this stage of education should have larger bearing on life.

(9) The syllabus of Mathematics for this stage of education should include Arithmetic, Algebra, Geometry, Coordinate Geometry, Mensuration, Dynamics, Statics, etc.

(10) The syllabus of the secondary stage of education should have two-fold purpose to achieve. On the one hand, it should prepare a student for higher education and, on the other, it should prepare him for life by creating living interest in the various articles around him.

CHAPTER XVII

TEACHING OF ARITHMETIC

Q. Describe briefly the importance of the teaching of Arithmetic.

Or

What are the two major and fundamental objectives of the teaching of Arithmetic ? What precautions should be taken in the method of teaching to achieve these objectives ?

Or

What is the importance of teaching Arithmetic in the schools ? What topics should be included and at what stage should they be taught ?

Answer.—"The need of a good command of Arithmetic by a skilled mechanic, by the up-to-date farmer, by the progressive professional man, by the successful merchant and by the efficient housewife, is so obvious as to need no discussion."

Value and utility of the teaching of Arithmetic.—While talking about the importance of the teaching of Mathematics and the causes of its being an important place in the school curriculum, we have seen that it has a lot of value in day-to-day life. This is equally true of Arithmetic, which is an integral part of Mathematics In fact, it is the Arithmetic which has greater value in day-to-day life. Most of the fundamentals of the numerical works and furniture etc. are guided by Arithmetic and its principles.

Apart from the practical value, Mathematics has cultural as well as disciplinary value. This was realised by the people of the ancient ages as well. In India and Egypt, we find the importance of the Mathematics even in the certuries before Christ. However, the same is true about the teaching of Arithmetic even today.

Major objectives of the teaching of Arithmetic.—Following are the two major objectives of the teaching of Arithmetic :—

1. Ability to perform various number-operations skilfully.

2. To provide variety of experiences, which will enable the students to apply quantitative procedure effectively in society.

The objectives may be split up and further divided. An attempt has been made to that effect in the following passages.

Certain general objectives of the teaching of Arithmetic.—Following are the general objectives of the teaching of Arithmetic :—

1. To lay foundation for the understanding of higher Mathematics.

2. To equip the students with the competence to enable them to understand the mathematical statements.

3. To develop the ability to have correct analysis and reach proper conclusions.

4. To have proper idea about the weights and measures.

5. To develop in the students the ability to perform computational jobs connected with the society with speed and accuracy.

6. To have a clear idea about the practical utility of Arithmetic.

7. To enable the students to make dependable estimates and approximations.

8. To develop resourcefulness and ingenuity.

General rules of teaching Arithmatic.—Teaching of Arithmetic, in order to be effective, has to be guided by certain rules and regulations. Unless these rules and regulations are properly put forward, the student shall not be able to understand things properly. Given below is an outline of the general rules of teaching Arithmetic :—

1. The teacher should *explain the general principles underlying the topic or the lesson* that is to be taught.

2. The teacher should *allow the students to work independently*. He should provide help, keeping in view the individual differences.

3. Proper exercise and drill should be given.

4. The teacher should do the correction work properly and in the best possible manner.

5. Attempt should be made to develop orderliness, accuracy and speed.

6. Student should be trained in oral Mathematics.

7. The interest of the students should not be allowed to diminish.

8. Students should be encouraged to do the written work systematically. They should be encouraged to do the rough work

in such a manner that it does not spoil the systematic written work.

9. They should be encouraged to write figures neatly, and correctly. Cancellations should be complete and clear.

10. The language of the work should be simple, understandable and correct.

11. The teacher should proceed from simple to complex and then come down to generalisation.

12. The examples that are given by the teacher to the student suit the circumstances of the students and they should also be understandable.

13. To Teach the students the value of orderliness.

Aims of the teaching of Arithmetic.—Nothing can be done aimlessly. Aimless teaching goes a waste. This is true about Arithmetic as well. The teaching of Arithmetic should aim at following :—

1. Development of mathematical thinking in the students.
2. Arouse interest in the quantitative side of world.
3. To impart knowledge about practical utility of Arithmetic in life.
4. Prepare students for the study of higher stages of Mathematics.

Different approaches in the teaching of Arithmetic.—In the teaching of Arithmetic different approaches have been talked about and emphasised upon. Given below are the *four approaches* that are generally employed in it :—

1. **Incidental approach.**—This is opposed to the traditional or drill approach. Here the teaching is done in a rational and incidental manner.

2. **Drill approach.**—In this method the teacher continues to repeat a thing before the students, till it gets fixed in the mind of the students. This is old formal approach.

3. **Social approach.**—In this approach, the experiences of the child occupy an important place. The teaching of Arithmetic is done in such a way that it is linked with the experiences of the child and his daily routine.

4. **Meaningful approach.**—In this method the entire programme of the teaching work is systematically worked out.

Fundamental thing to be kept in mind while teaching Arithmetic.—One basic thing that has to be kept in mind while teaching Arithmetic is that it is 'a means of developing the child's natural ability and so the teaching should lead to the methods or the results that bring about accuracy and speed. The best thing in the teaching of Arithmetic is that too much of emphasis should not be laid on theory. The students should be made to do things in a practical manner and their ability should be tested.

Place of drill in the teaching of Arithmetic.—Teaching of Arithmetic cannot be successfully done unless a good deal of drill is provided to the students. Drill enables them to have the repetition of the knowledge which ultimately strengthens their experiences. Drill is nothing but exercise. What is needed is a careful and systematic planning of the exercises and the work. Attempt should be made to co-relate the drill of the exercises with the facts of life. Such a correlation will have a lasting effect. Following things are to be kept in mind in regard to exercises and drill :—

1. It should be clear and vivid.

2. It should have association with the past and attempt an association in future.

3. There should be the element of motivation. Unless it is there, the students shall not take interest in it.

4. There should be no scope for guess work.

When should the teaching of Arithmetic start ?—We have already discussed the importance of Arithmetic. This discussion makes perfectly clear that the teaching of Arithmetic should start at the pre-primary stage or infant stage. An infant should be made to learn things about Arithmetic through playway method. In other words, it means that the teaching of Arithmetic should start along with the beginning of education.

Importance of number in the teaching of Arithmetic—Number is the basic element of Arithmetic. Without number, nothing is possible in Arithmetic. It is, therefore, necessary that the teaching of Arithmetic should start with number. At the infant stage, the students should be made to learn about numbers, their different combinations, and each methods of numeration. Once they have learnt about the methods of representation of number, they shall be able to proceed successfully.

Teaching of addition in Arithmetic.—The first stage of the application of numbers is addition. In this process the numbers are put together and their result is found out. In the teaching of Arithmetic it occupies an important place. This should start at the pre-primary stage and go on up to primary stage. However, this proces shall go on even in the higher stage of education, but the teaching of addition as a topic should not go beyond the primary stage of education.

First of all, the students should be trained in the counting of numbers. Then they should be trained in the combination of numbers. This may be done in a graded and systematic manner. First of all, the students should be trained in the addition of one digid, then two digids, then three digids, and so on and so forth. In the teaching of addition, the teacher should start from simple to complex.

The process may be explained with the help of the figures and other concrete objectives such as pebbles, pieces of wood, inner objects of ripe tamarind etc.

While teaching addition, the students should be told about the correct procedure of its working.

Oral work in the teaching of addition.—Oral work occupies an important place in the teaching of Arithmetic. It has its importance in the teaching of addition as well. Through oral work, it is possible to do things more accurately and give them proper practice. Given below is an example of teaching of addition of three digids and more :—

$$\begin{array}{lclcr}
\text{(A)}\ 342 & = & 300+40+2 & \times & 700 \\
+\ \ 457 & & 400+50+7 & & 90 \\
 & & & & 9 \\
\cline{3-3}\cline{5-5}
 & & 700+90+9 & & 799
\end{array}$$

Teaching of substraction.—In addition, the pupils are to be taught about putting numbers together, but in substraction they are taught to take out numbers from one another. Only smaller number can be taken out from a bigger number. Teaching of substraction cannot go beyond the primary stage of education. It should begin at the pre-primary stage and end with the primary stage.

Proceed from simple to complex.—The teacher should proceed from simple to complex. First the students should be given substractions of one digid and later on, of larger digids.

From concrete to abstract.—In the beginning, the teacher should take help from concrete things, such as balls, coloured pieces of wood, beeds, etc., and teach substraction to the students.

Through this method, the system of borrowing and returning may also be taught.

While teaching substraction, the method of changing coins may also be explained. This would help them in practical life.

Importance of oral work.—In the teaching of substraction, oral work occupies an important place. In the beginning, students should be taught to do substraction orally and then in the written form.

Teaching of Multiplication.—It is a shortened method of 'addition of equal quantities.' In this method there is addition of more than one group. It is a quick method of addition. It helps and simplifies the process of addition. It is, therefore, advisable to teach it at the primary stage of education and then take it in more difficult and complex form at Junior High School or pre-secondary stage of education. In the beginning the students should be made to learn the multiplication tables of 2's, 5's and 10's. Then they should be made to learn multiplication tables of 4's, 8's 3's 6's and 9's. In the end, the multiplication of 7's should be taught. Learning of multiplication tables is very helpful in multiplication.

Steps of teaching of multiplication.—In order to teach multiplication successfully, the teacher should take up the following steps :—

1. He should proceed *from simple to complex*. In other words, in the beginning, the multiplication of simpler digits should be taught, and then of the complex digits.

2. First of all, multiplication of one digits may be taught and then the multiplication of the digits that have carry over.

3. While teaching multiplication, the knowledge of multiplication tables of students should invariably be put to test.

4. While teaching multiplication, the students should be explained that in multiplication any of the digits may be used as a multiplier and the result shall be same. For example, $7 \times 8 = 56$. Similarly, $8 \times 7 = 56$.

5. While teaching multiplication, the teacher should determine the difficulties and the drawbacks of the students. If certain basic difficulties are removed, it shall be possible to teach them the process quickly and scientifically.

6. **Drill and exercises.**—In order to strengthen the knowledge and experiences, it is advisable to give them more of practice and drill in multiplication tables.

Teaching of division.—It is an easy and shortened way of substraction. In fact, division is nothing but substraction in groups of more than one. It is a process, which is reverse of multiplication which means addition of groups of more than one.

Fundamentals of the teaching of division.—While teaching division, the following things should be kept in mind. :—

(*a*) **No rote memory but assimilation by repeated exercises and verification.**—The students, while they are being taught division, should be discouraged from using rote method. They should be encouraged to discover and assimitate the rules. This can be done by giving them repeated exercises and verification.

(*b*) **Concrete problems.**—Teaching of division should be done with the help of concrete problems. It should not be done by formal or abstract methods.

It may go with division, substraction, etc. Some people have suggested that in the begining substraction and addition should be taught and then division and multiplication, etc. should be kept up. In is not a very correct process. All these methods may go together. This would help the students to have a variety of the experiences and know the difference between different methods.

(*c*) **Practice in different situations.**—Students should be made to practise the rules in different situations. If a particular rule of division has been taught, they should be made to practise it in different situations. Such a thing would help them to strengthen their knowledge and experience.

In the beginning, divisions without carry over should be taught and then the complex ones with carry over should be taken up.

(*d*) Students be encouraged to write down the answer in the form that they may write down the dividend and the remainder.

Teaching of fractions.—Use of fraction is more to be found in Algebra than in Arithmetic. But the base for it is to be provided by Arithmetic. If the student is able to acquire proper knowledge of the fractions in Arithmetic, he is able to solve such sums of Algebra quite easily.

Fraction is nothing but a part of a whole. While teaching fraction, the students should be told that if the numerator and the denominator are divided by the same figure, the result shall be the same. It can be explained by the following example :—

$$\frac{3\times 2}{5\times 2}=\frac{6}{10}=\frac{3}{5}$$

$$\frac{3 {-}{-} 2}{5-2}=\frac{3}{2}\times\frac{2}{5}=\frac{3}{5}$$

Things to be explained in the teaching of fraction.—The teaching of Mathematics requires the explanation of the following technical terms :—

1. Numerator.
2. Denominator.
3. Proper fraction.
4. Improper fraction.
5. Reduce.
6. Invert.
7. Substracting the same number from the numerator and Denominator.
8. Multiplying the fraction with the reverse numbers.

Definition—*Fraction* is a division or a part of a unit or a whole.

Denominator is the devisor or the number which is below the line in the fraction. It indicates the number of the parts into which the unit or the whole is to be divided.

Numerator is the dividend or the number above the line in fraction. It indicates the number of parts into which the unit is divided.

Proper fraction is that whose numerator is less than its denominator as $^5/_6$, $^6/_8$, etc.

Improper fraction is that in which the numeration is equal to or greater than denominator, such as $^3/_3$ or $^8/_4$ or $^6/_2$.

Difficulties in understanding fraction.—It is, of course, a difficult item to be learnt. The students should be taught to learn them scientifically and properly.

They can be made to learn only if things are explained to them in proper manner. This item can be taught in the highest classes of the primary stage of education and then in the beginning stages of the pre-secondary or junior high school stage of education.

Decimal fractions.—In our country, now decimal fractions occupy an important place. Our coins are based on this principle. The same is true about the weights and measures. The decimal fractions have, therefore, to be understood properly.

It is not a very difficult task to learn about decimal fractions. It is, in fact, a fraction whose denominator is never written and the unwritten denominator is limited to some power of 10. It is easier to learn the decimal fractions.

Division, Multiplication, Subscription, Addition, etc., of the decimal fractions.—We may have sums relating to decimal fractions about addition, substraction, multiplication, etc. All these sums should be properly handled and taught. The figures at the right land of decimal, from students view, present ·1, ·01, ·001, etc and the left present the whole numbers, beginning from the first letter, which is called unit

Utility of teaching decimal fractions in India.—As already stated, Indian coin is now based on decimal system. Now our rupee has 100 equal parts of one paisa each. The teaching of decimal fractions is, therefore, quite important.

This is true about different countries as well.

Measurement.—In our country, now there is metric system of measurement as well as weight. All these things have the bearing on decimal fractions. This thing has to be taught, therefore with greater caution and keeping in view its utility to the people of the country.

Ratio.—It is a relationship of one quantity to another. It is indicated like this—

6 : 1

Ratio of one number to another is nothing but the quotient of the first number divided by the second. The two numbers, that are used in a ratio, are called the terms of the ratio. The first one is called *Antecedent* and is the dividend. The second digid or number is called *Consequent* and is the devisor.

Teaching of Ratio.—Normally it should begin at the Junior High School stage of education, *i.e.*, from class VI and should go on up to class VIII. In secondary classes more complicated sums of ratio may be taught and explained.

In the beginning, the teacher should teach this tapic or subject with the help of concrete things, such as balls, beeds, etc. Then, he should proceed in an abstract manner and teach it with the help of the figures. In other words, the teaching should proceed from simple to complex.

The experience or the knowledge should be strengthened with the help of the variety of examples, drills and exercises.

Ratio is indicated by different signs such as 3 ÷ 4 or 3/5 or 3 : 5.

Q. The students may be asked to do the exercises as given below :—

Divide Rs. 100/- between two boys in such a way that the younger one gets 2 as compared to the 4 of the elder one. In other words, the division should be in the ratio of 2 : 4.

Proportion.—Ratio and proportion are, in fact, closely interlinked. For the understanding of one, the knowledge of the other is inevitable. When 4 quantities are so connected with the ratio of the first to the second is equal to the ratio of third to the fourth it is suggested that they are in proportion. In fact, it is a kind of relationship and that is why the order of sequence of terms of ratio is pretty important.

The four figures or the four terms in proportion are termed like this.

The first and the fourth number or digids are called *extremes*, and the second and the third are called means or middle terms. It may be illustrated like this—

5 : 20 : : 4 : 16

In this arrangement, 5 and 16 shall be called extremes and 20 and 4 shall be called means.

Proportion—simple or compound.—Proportion may be simple as well as compound.

Direct and Inverse.—Proportion may be direct as well as inverse. In direct proportion, both the figures rise and diminish simultaneously. While in inverse if one number goes up the other goes down.

The teaching of proportion should be taken up at the Junior High School or Middle stage of education. More complicated sums may be taken up at the High School stage of education.

Percentage.—It is an important aspect of the teaching of Arithmetic. It has bearing on different aspects of the subject. It is used in interest, price, profit and loss, etc. As the word indicates, per cent means per hundred. It is a part of the 100 as a whole. It is indicated like this, %. It has a bearing on decimal as well. Once the decimal has been understood, it would not be difficult to calculate percentage.

Teaching of percentage should also start at the Junior High School stage of education and go in a complex form up to the High School stage of education. Here the teacher should again proceed from simple to complex. This is the most psychological method of teaching of the subject.

Interest—simple or compound—Percentage has an important bearing on the question of interest. Interest, as we all know, is surcharge on money lent or money-giving. It is of two types :—

(*a*) Simple ; and

(*b*) Compound.

Simple interest is an addition to the capital in the simple form. For example, simple interest on Rs. 100/- at the rate of 5 % per annum shall be Rs. 5/- only. The formula for finding out simple interest is given below :—

$$\text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

The teaching of simple interest should start from simple to complex. In other words, in the beginning, the students should be taught about simpler questions on simple interest and then difficult sums may be taken up.

Teaching of this topic should start at the Junior High School stage of education and go up to the High School stage.

In compound interest, the interest is also charged on the interest that falls due at the end of the year. In other words, if somebody has taken a loan of Rs. 100/- at 5 % of interest, at the end of the year the amount shall grow 100+5 and then the interest shall be charged not on Rs. 100/- but on Rs. 105/-.

Compound interest should be taught after teaching simple interest. The formula for finding out the compound interest is like this :—

$$\text{Compound interest} = \text{Principal}\frac{(1+\text{rate})}{100}\text{time} - \text{amount given out}$$

as principal.

Discount.—This too has the bearing of the percentage. It is, in fact, a reduction in the diversed price or the price at which the thing is to be sold. Is is, in fact, the present worth of the actual amount. Discount is equal to interest on present worth.

Profit and loss.—Profit and loss is the principle of every day life. Students are well aware of every day sale and purchase of things. They should, therefore, be told this in the following manner :—

Profit or gains is the selling price minus cost price.

Similarly, loss is the cost price minus selling price.

In the profit, selling price is more than the cost price but in loss, selling price is less than the cost price. Sometimes the question of percentage is also involved.

Sums on profit and loss should start at the Junior High School stage of education and go up to secondary stage of education. In teaching the sums of profit and loss, the teacher should proceed from simple to complex or from simple to difficult. He should first teach the students the sums that are simple and then he should proceed to difficult sums.

Average.—The teaching of average may be introduced with help of certain experiences about average in the life of the students, such as a game of cricket. Then, with the help of this method, the students may be taught about average of rainfall, temperature, etc. With the help of the correlation of the knowlege of Geography, it may be possible to teach this thing easily and without much difficulty.

While teaching the average portion of Arithmetic, the teacher should proceed from simple to complex. This topic should be introduced at the Junior High School stage of education and taken up to higher secondary stage of education. In fact, average is the middle number between the smallest and the largest number. It is, in fact, an indication of generality. If certain numbers are arranged in order, the average shall be somewhere about the middle number.

The teacher may introduce the subject with the help of some concrete things like pieces of wood or drawing lines on the blackboard.

Stock and share.—When more than one person join a particular business and form a company, they all subscribe towards the capital. Each individual's contribution towards capital is called the share and those who contribute are called shareholders. *Share* is generally the capital divided into number of equal portions. The profit is divided into the shareholders in proportion to the shares they possess. In such cases, the profit is called dividend.

Normally, it is not possible for a shareholder to recover the amount of share from the company. He can, however, sell his share to another person. In such a situation, the purchaser of the share shall have the right to receive the annual dividend.

There is fluctuation in the price of share. If the company is prosperous, the price of the share goes up, but, if the company is a losing concern, the price of the share goes down.

Stock is the capital of a company.

Questions on share and stock should be taken up along with the sums of profit and loss. This topic should be introduced at the junior high school level and more difficult sums should be taken up at secondary or higher secondary stage of education.

Area.—While teaching area, the teacher should try to start with the concrete example. The teaching of this topic should not start with the theoretical knowledge. Some figure should be drawn on the black-board and with the help of that figure the concept of area should be made clear to the students. They should then be told that area is equal to length × breadth and volume is equal to length × breadth × height. The area is written in squares or the words square meter or square centimeters are used after the figure of area. Students should be explained about this thing as well. Area is in squares.

The topic of area should be introduced at the junior high school level and should go up to higher secondary stage of education. The teacher should start the simpler sums and then more difficult and complicated sums should be taken up.

Volume.—Volume is equal to length × breadth × height. They should not be told in form of a definition to the students. They should be given a concrete idea about it. The figure or the thing whose volume is to be found out should be divided into several units. Then on the basis of those units, the volume of the figure may be found out. Normally volume is indicated by cubic measures. If the volume of a particular object is 27 meters, it should be written as 27 cubic meters.

While teaching the sums about volume, the teacher should again start from simple to complex. The basic object is to make the concept of the volume clear to the students. The process of the classification of the concept of volume should start at junior high school stage of education and it should go up to the higher secondary stage.

Squareroot.—This is more applied in Algebra. But its basic concept lies in Arithmetic. The object of the teacher should be to make the students perfectly clear about the squaring or finding out the square or squareroot of a particular number.

In finding out squareroot, it is not necessary that the squareroot may be in a positive figure. It may also be in negative figure, such as—3.

Teaching of squareroot.—Here again the teacher should not try to be formal and theoretical. What is required is the clarification of the idea of squares and squareroot. Here geometrical method may be applied and a particular figure may be drawn on the black-board. That figure may be divided into different squares and on the basis of that division, the squareroot may be found out. This would help the teacher to make the subject perfectly clear to his pupils.

The teaching of this topic should start at the junior high school stage of education and go up to higher secondary stage. The principle of proceeding from simple to complex should also be applied here. Without this, it shall not be possible to make the concept clear.

Q. How will you introduce the idea of numbers and four simple rules with their elementary operations in infant schools ?

Right methods are of utmost importance in the teaching of Arithmetic to infants. The chief aim in teaching Arithmetic in an infant school should be to train children to see the simplest relation between quantities and numbers and to deal readily with small numbers. Notation of number should be acquired through practical work and experience, the principle of 'learning by doing' must find an important place in teaching arithmetic to children at the infant stage, and the senses of sight and touch should be exercised in the discovery of numerical relations by them.

In teaching counting, the recitation and chanting of nursery rhymes dealing with numbers has great attraction for children, especially if these are accompanied by actions. In counting, the teacher should aim at correlating the impression gained by the pupils through the sense of sight, touch and hearing. The symbols representing numbers must be connected with the word and the figure which enable us to recognise it.

(*i*) The figure '2'

(*ii*) The written and spoken word 'two'

It is very useful to take the aid of the Montessori apparatus for the teaching of numbers.

The elementary operations of numbers can be given with the help of fruits, sticks etc. When they have to take 25 beads, they will take two bags of 10 beads and 5 loose beads.

In addition, suppose we want to teach them $5+2=7$, ask them to take 5 balls and 2 balls separately. Then counting them collectively, they will come to know that 5 balls plus 2 balls are equal to 7 balls. Thus, they will know $5+2=7$.

Similarly, in case of subtraction, multiplication and division, the same apparatus can be used.

Multiplication is repeated addition and division is repeated substraction.

Q. How will you further develop the operation of numbers ?

In teaching addition and subtraction tables, it has been found successful to allow children to experiment with sticks, beads, stones etc. The pupils should be given practice in reading numbers, in setting them down in the column of units, tens and of hundred and in writing them in words. The nature of these numbers may be demonstrated by using bundles, each of which contains tens of articles or counters The pupil may be encouraged to frame their own problems arising out of the things they are interested in. Sufficient practice should be given in the addition of single digited figures in vertical columns and in horizontal lines. The first step in adding two digited numbers should be by means of examples such as :—

$4+3=?$ $4+?=7$ $\begin{array}{r}7\\3\\\hline ?\end{array}$ and so.

Four Simple Rules

1. The four simple rules are : (i) Addition, (ii) Subtraction, (iii) Division, and (iv) Multiplication.

2. These are nothing but four different ways of counting.

(i) Adding is counting forward.

(ii) Subtraction is counting backward.

In multiplication or division, we count forward or backward by leaps of uniform length.

The idea of Addition or Subtraction can be imparted with the help of Montessori apparatus.

Students must be first familiar with natural numbers and numbers such as units, tens, hundreds, thousands etc.

This can be done by Fingers and Toes.

Idea of Addition and Subtraction should be imparted at the same time, one being of inverse of other.

Examples :—

$$10+6=16$$

16 is greater than 10 by 6.

10 is lesser than 16 by 6.

If we count from 10 uptil 16, we find that difference between 16 and 10 is 6.

So these are inverse processes.

1. Always subtract from concrete quantities.

Example :—

1. 4 Books+5 books=9 books i. e. $4+5=9$

3 pens +2 pens=5 pens i. e. $3+2=5$

2. First of all, take those numbers which do not involve carrying over.

Cows	Goats	Rs.	As.	Tens	Units
3	6	6	8	4	3
7	3	3	1	5	6

2. Idea of similar quantities can be classified in Pies to Pies, As. to As., Rs. to Rs., Units to Units, Tens to Tens and so on.

3. Verification can be done as

Vertical columns=6

$$\text{Horizontal}=6+3=9.\frac{3}{9}$$

$$7+9=16$$

$$=(7+3)+6=10+6$$

$$=(9+1)+6=$$

Example :

$$\begin{array}{cc} \text{Tens} & \text{Units} \end{array}$$

$$\left.\begin{array}{cc} 3 & 7 \\ 2 & 2 \end{array}\right\}\text{Vertical columns}$$

$$\begin{array}{c} \hline 59 \end{array}$$

$$\left.\begin{array}{l} 33 = 3 \text{ tens} + 7 \text{ units} \\ 22 = \underline{2 \text{ tens} + 2 \text{ units}} \end{array}\right\}\text{Horizontal columns}$$

$$5 \text{ tens} + 9 \text{ units} = 59$$

Involving carrying over

$$\begin{array}{l} 57 = 5 \text{ tens} + 7 \text{ units} \\ 5 = \underline{\qquad\qquad 5 \text{ units}} \\ \qquad 5 \text{ tens} + 12 \text{ units} \\ \qquad\qquad (10 + 2 \text{ units}) \\ \qquad\qquad = 1 \text{ ten} + 2 \text{ units} \\ = 6 \text{ tens} + 2 \text{ units} \\ = 62 \end{array}$$

Subtraction

Idea can be best imparted with the help of beads, sticks, balls and frames etc. It should work simultaneously with addition. It can be well demonstrated with the help of a set of test cards *i. e.*

$$5 + ? = 8 \qquad 9 + ? = 16$$

$$\left\{\begin{array}{l} 6 - ? = 4 \\ 5 - ? = 2 \\ 7 - ? = 3 \end{array}\right\} \qquad \left\{\begin{array}{l} 6 - 4 = ? \\ 5 - 2 = ? \\ 7 - 3 = ? \end{array}\right\} \qquad \begin{array}{l} ? - ? = 4 \\ ? - ? = 2 \\ ? - ? = 3. \end{array}$$

Method of Complementary Addition

2. First of all start with an addition sum.

$$\begin{array}{r} 578 \\ 386 \\ \hline 984 \\ \hline \end{array}$$

Rub out the second lines thus :

$$\begin{array}{r} 578 \\ \rightarrow 386 \\ \hline 964 \end{array}$$

To find the rubbed number.

i. e., 8 and 6 make 14, 8 (*i.e.*, 7 and 1) and 8 make 16.

6 (*i. e.*, 5 and 1) and 3 make 9.

Usual form of example :

$$\begin{array}{r} 138448 \\ 54683 \\ \hline 83765 \\ \hline \end{array}$$

3 and 5 make 8
8 and 6 make 14
7 and **7** make 14
5 and 3 make 8
5 and 8 make 13.

5. Method of Taking away

Two methods are in use

First is a method of equal additions.

Second is a method of Decomposition (greatly used) as for example :—

$$\begin{array}{r} 14 \\ 58 \\ \hline \end{array}$$

Units 8 from 4, you cannot

∴ 8 from 14 = 6.

Tens : By equal addition	By decomposition
6 from (5 + 1) from 9 = 3	5 from 8 = 3.
Difficult to explain.	Much in vogue.

Subtract 367 from 845 by decomposition

$$845 = 800 + 40 + 5 = 700 + 130 + 15$$

$$367 = 300 + 60 + 7 = 300 + 60 + 7$$

$$= 400 + 70 + 8$$

$$= 478.$$

By Equal Additions

$$845 = 800 + 40 + 5 = 800 + 130 + 15$$

$$367 = 300 + 60 + 7 = 400 + 60 + 7$$

$$400 + 70 + 8$$

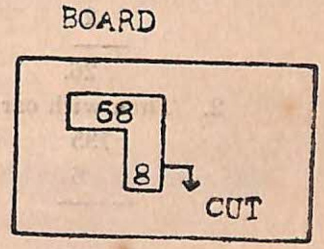
$$= 478.$$

1. This method requires accuracy, skill, exactness and speed. Hard for the child to understand.

2. Verification *i. e.*, addition by subtraction and subtraction by addition should be there in such like questions.

According to J. W. A. Young, the idea of four simple rules should be departed with such planned board and paper,

68	47	29	31	8
23	32	10	6	70
32	12	58	9	67
34	56	78	9	10



by moving the cut paper on the table.

MULTIPLICATION TABLES

Multiplication tables have been defined as shortened addition of equal quantities. These are very helpful in the following order :—

i. e., first 2'5, 5'5, 10'5
then 4'5, 8'5
then 3, 6, 9'5
and last of all 7'5

They are of great use if connected with one another.

Table should be built by the students themselves with the help of apparatus. Tables are also of a great use in daily life. Their use sould become automatic by drill and repetition. They should be made mechanical but should not be mechanically made.

These can also be taught with the Commutative law such as

$$\left.\begin{array}{c} 2\times9=18 \\ \text{or} \\ 9\times2=18 \end{array}\right\} \text{Products of any two nos. is always the same.}$$

MULTIPLICATION

1. It is an addition in groups of more than one.

14→Multiplicant
2→Multiplier
28

2. First with multiplicant of up to tens
    1. and multiplier units

13
2
26.

2. Then with carrying over.

735	547
6	8

3. Multiplying by 10 or 100 should be realised by actual practice that it is nothing but to add zeroes.

4. Always multiply by the L. H. figure of the multiplier.

	OR→This better
9573	9573
3058	3058
28719***	76584
47865*	47865*
76584	28719***
29274234	29274234

5. Compound Multiplication of Prof. Method Num. Multiply

£	$	d.
15 —	14 —	9 by 365
365	365	365
365*	365*	12/3285
1825	1460	
269	273←	$273—9d.
£5744	20/5383	
	£269—3$	

Ans. £5744—3$—9d.

DIVISION

1. It is subtraction in groups of more than one.
2. It is an inverse process of multiplication.

$$i.\,e., \frac{6}{2}=3$$

$$6 = 2\times 3$$

3. How to impart the idea of division
   1. Through concrete examples, *i. e.*

   If 12 apples are shared among 3, how many each gets.

   i. e., $12 \div 3 = 4$

Ist	II	III
00	00	00
00	00	00

4. Multiplication and Division.

Multiplication	$6\times 4=?$	$24\div 4=?$	Division
	$7\times 5=?$	$35\div 5=?$	
	$8\times 6=?$	$48\div 6=?$	
	$9\times 7=?$	$63\div 7=?$	

5. Partition is really a fraction

6. Simple Division 49769 by 253.

7. Compound Division.

Prof. Num :—

```
                     17
      ------------------------------------
73)  £                $                d
   1290               2                3
     73              980              396
                     ---              ---
                  73)982(13        73)399(5
                      73
   ----              ---               365
    560              252              ---
    511              219               34   remainder
   ----              ---
     49               33
     20               12
   ----              ---
    980              396
```

POINTS FOR REVISION

1. Teaching of Arithmetic is very important in the teaching of Mathematics.

2. Teaching of Arithmetic has utilitarian as well as cultural value.

3. Following are the two objectives of the teaching of Arithmetic :—

(*a*) To develop in the learner the ability to perform various number-operations with skill and understanding.

(*b*) To provide the pupils with rich variety of experiences that would enable them to carry out quantitative procedure effectively in their actual life.

(*c*) To teach them that systematic working always pays in arithmetic and in life.

4. The teaching of Arithmetic should be guided by the object of imparting the knowledge about computation and numbers to the students.

5. Following are the *different approaches* of the teaching of Mathematics :—

(*a*) Incidental approach.

(*b*) Social approach.

(*c*) Drill approach.

(*d*) Meaningful approach.

6. The teaching of Arithmetic should start at the infant or the pre-primary stage of education. The teacher should teach the principles in playway method.

7. In the teaching of Arithmetic, as far as possible, rote memory should be avoided. Attempt should be made to make the idea clear.

8. Multiplication table is a fundamental requirement of the successful application of the principles of Arithmetic. The students should be made to learn this thing at the primary stage of education.

9. In the teaching of Arithmetic, the principle of proceeding from simple to complex should always be there.

10. The teacher should be more interested in making the concept clear than giving the formal education not only abstract and theoretical knowledge.

11. The teaching should start from addition, then it should proceed to substraction, multiplication, division, fraction, etc.

12. Once these simple things have been made clear, questions on decimal fractions, ratio, proportion, partnership, percentage, profit and loss, average, simple interest and compound interest, discount, stock and share, area and volume, squareroot, etc. may be taken up.

13. The teaching of Arithmetic should not go beyond the higher secondary stage of education.

14. Elementary Arithmetic should be taught at the primary stage, then difficult one at the Junior High School stage and then more difficult one at the higher secondary stage.

CHAPTER XVIII

TEACHING OF ALGEBRA

Q. Describe briefly the importance and the requirements of the teaching of Algebra in higher socondary schools ?

Or

"Algebra is Arithmetic generalized". What do you mean by this statement ? Describe briefly how will you introduce the first lesson of Algebra to the students ?

Or

What are the difficulties that the students face while learning Algebra ? What method should be applied to do away with these difficulties ?

Or

What are the aims of teaching of Algebra in the schools ? How far is it possible to achieve these aims with the help of the curriculum at the different stages of education ?

(L. T.)

Answer.—Algebra-defined.—The word Algebra has been derived from the Arabic world '*Al-Jebr,—Al-Muqabulah.*'—In Arabic system of Mathematics, Algebra was a device of solving the sums by force. Those sums that could not be solved with the help of Arithmetic were solved with this process. With the passage of time, Algebra has assumed new dimensions and a new concept. *It presents a radical new and different approach to the study of quantitative relationship, characterised by a new symbolism, new concept, a new language. A much higher degree of generalisation and abstraction, then has been encountered previously, by the fact that in contrast to Arithmetic, Algebra is more concerned with the conscious examination and study of processes than with particular answers to particular problems ; and by the essential dissociation of many of its parts from intuition and concrete experience.*"

Sometimes Algebra is called '*generalised Arithmetic*'. It is a method in which abstract numbers represent the numerical measures.

Aims of the teaching of Algebra.—The teaching of Algebra is, in fact, complementary to the teaching of Arithmetic as well as Geometry. It has the following aims which speak of its importance :—

1. **Removal of ambiguity and deficiencies of language.**—It tries to present the numbers and fiigures in a method, which is free from ambiguity and deficiencies of a language.

2. **Helps solution of difficut problems of Arithmetic.**—Algebra provides the students with the methods through which the students may solve even difficult problems of Arithmetic. Factors and equations are such methods. They save labour and economize energy.

3. **Method of presenting abstract relationship in new language and new symbols.**—Algebra provides a new approach in which the abstract mathematical relationship is presented in new symbolism and new language, which is more concrete and understandable.

4. **Simplification of calculation.**—Through Algebra it is possible to help simplification of calculation and to save time and energy.

5. **Practical value.**—It has a practical value in different aspects of life. It trains mind in such a way that the training may be applied in different aspects of life.

6 **Satisfactory checking of results.**—Through Algebra, it is possible to check the results in a satisfactory and simple manner.

7. **Use of inductive method.**—Teaching of Algebra lays emphasis on inductive method. This is more concrete and helpful.

Importance of the teaching of Algebra.—Algebra has a bearing on Arithmetic as well as Geometry and so it is useful to both these branches of Mathematics. It has the following advantages :—

1. It may create concrete interest in the students.

2. It may induce them to study certain topics.

3. It may provide them with better opportunities for study of certain topics.

When should the teaching of Algebra begin ? The teaching of Algebra should begin at the Junior High School stage of education. Its beginning should be formal. It should be concrete. While beginning the teaching of Algebra, attempt should be made to corelate it wtih Arithmetic. If certain sums of Arithmetic are solved through the method of Algebra, it shall be more convenient for the students to understand it.

Importance of drill and exercises in the teaching of Algebra.—In Algebra, drill and exercises also occupy an important place. The students should be given a good deal of exercises to strengthen their knowledge.

Methods of teaching of Algebra.—Normally, in the teaching of Algebra, following three methods are employed :—

1. Formal method of the teaching of Algebra. Addition, substraction, multiplication and division be taught as in the arithmetic, simply the letters occupy the place of numbers.

2. **Equation and problem method of the teaching of Algebra.**-In the teaching of Algebra, it is not possible to distinguish the two methods and use them separately. Both these methods sometimes combine. In the beginning problem and equation method is to be employed. If a problem is posed before the students, they shall be able to develop interest in the subject.

3. **Teaching of Algebra and literal number of quantities.**—In Algebra, instead of numbers, certain letters or literal numbers are used. These numbers are, in fact, abbreviation of certain words. In the teaching of Algebra, these things should be clearly explained to the students.

Order of operations.—In the teaching of Algebra, certain operations have to be conducted. First of all, multiplication is to be taken up and then the division. Before that, addition and substraction should be done.

Use of · **brackets.**—Brackets occupy an important place in the teaching of Algebra. It is very important to explain about the use of the brackets and the method of their removal. It is necessary to explain the following things in regard to bracket :—

(*a*) The students should be given a clear idea and understanding of the meaning of the bracket.

(*b*) They should be trained as to interpret correctly mathematical expressions in which the brackets are used.

(*c*) They should also be trained to employ the brackets correctly. They should know when the brackets are to be used and when they shall be helpful.

(*d*) They should also be trained to transform expressions involving brackets into equivalent expressions that do not have brackets.

(*e*) The terms that are enclosed within a pair of brackets are one quantity like a packet of biscuits or a bundle of boxes etc.

(*f*) Brackets are like containers or the limits or areas that confine certain things. The students should be trained that the removal of bracket is nothing but removing the container or doing away with the area or limit.

Brackets are used in multiplication, area, addition, etc.

Teaching of equation.—It is an important element in the teaching of Algebra. In fact, 'equation is a symbolic expression showing one expression equal to another'. Genrally there is a sign of = between two such equations. It is very helpful in the solution of the problems. It is like a sum of Mathematics. In fact, it is a complete sentence in language.

Teaching of equation to the beginner.—We have already seen that the teaching of the solving of the problems in Algebra should begin from the teaching of equation. While teaching equation, attempt should be made to pose the problem before the students.

Four Axioms of the teaching equations.—While teaching equation, the following four axioms should be kept in mind :—

1. "If equal quantites are added to equal quantities, the results are equal."

2. "If equal quantities are substracted from equal quantities, the results are equal".

3. "If equal quantities are multiplied by equal quantities, the results are equal".

4. "If equal quantities are divided by equal quantities, other than theory, the results are equal".

Problems and equations.—In the problems of Algebra, certain statements are translated into algebraic numbers and statements. The teaching of equations and problems should start from translation of such statements into algebraic figures. Then the procedure should go on.

Directions to the students for solution of problems of Algebra.—In order to help the students, to solve the problems of Algebra, the students should be given the following directions :—

1. They should be given specific problems.

2. Then, they should be asked to translate the unknown number or numbers into algebraic language *i. e.* suppose a letter in its place.

3. The students should be asked to find out two expressions or equal quantities. Then, these equal expressions should be connected by equal signs that form an equation.

4. Then, they should be asked to solve the equations for unknown letters.

5. In the end it should be checked that the answers satisfy all the conditions of the problem.

Identical equation or identity in equation.—When the values of a particular letter are always equal and an equation like that is presented, it is called an identical equation. Given below is an example of the same :—

$$2(x+4)=2x+8. \ x=10$$

Conditional equation.—In conditional equation, there is only one or a limited number of values. In such an equation, there is a condition and, therefore, it is called a conditional equation. Given below is an example of the same :—

$$3x=3+4=13.$$

It shall be true only when $x=3$. In other conditions it shall not be true. Here the condition is that x has the value of 3.

Different types of equations.—Other than the equations given above, equations are also of the following types :—

1. Linear Equation.
2. Quadratic Equation.

Graphical method of solving problems or equations.—In this method, the solution of the problems or the equations is found out by plotting the figures on the graph. It is not a general method, but it is used. It is more used in factors and squareroots.

Directed numbers or signed numbers.—Numbers are generally of two types :—

(*a*) Directed numbers ; and

(*b*) Negative numbers.

Directed numbers are those when the signs of + and — are used to show direction sense. They are different from the signs of operations of addition or substraction.

Negative quantities and numbers are those when these quantities or numbers possess not only magnitude but also sense, direction or order.

Teaching of the directed and the negative numbers.— Students have to be taught about these numbers. In fact, in Algebra these things are quite important. First of all, the teacher should introduce the directed numbers with the help of concrete examples. Then, he should explain about the negative numbers. It can again be introduced by concrete example of a line where the centre may be marked theory and on the left may be given negative numbers and on the right positive numbers.

Manipulation of the signs of '+' and '−'.—In Algebra, unlike Mathematics, signs of '+' and '−' do not only indicate addition and substraction. They indicate certain other things as well. They sometimes indicate time, sometimes rise and fall etc. It is necessary to explain these things, to the students, so that they may manipulate these signs properly.

Teaching of addition in Algebra.—It is, in fact, an operation with directed numbers. It should begin with concrete examples or problems. A line may be drawn and a centre may be marked as zero. On the left may be given negative numbers, and on the right positive numbers. Then, the solution of the problem may be taken up. When plus quantities are to be added, movement should be to the right and when minus quantities are to be added, the movement should be to the left of the sign of equality.

Teaching of substraction in Algebra.—It is, as compared to addition, more difficult to learn. Here the concept has to be more clear as compared to Arithmetic. It is, as we all know, the reverse process of addition. An example is given below :—

Take away things from a group of things. The problem $(-5)-(-2)$ can be interpreted as removing two minutes from a group of five minus ones. It may be represented like this—

$$(-1, -1, -1) \text{ Or } -3$$

In the teaching of substraction, the teacher should start from simple to complex. If he does so, he will be able to make the students learn the subject scientifically.

Multiplication.—After teaching addition and substraction, the teacher should proceed to multiplication in Algebra. '×' is a sign of operation and multiplication in Algebra.

Teaching of factors.—The teaching of the factors should not start before class VIII. It is a more difficult and complicated process.

Factorization is a process that enables the people to perform, in most effective manner, certain algebraic processes with fractions, radicals and equations. It also enables the students to understand the ways in which algebraic expressions may be combined or decomposed.

The teaching of fractions should follow the teaching of mulitiplication. It is, in fact, a difficult portion of Algebra.

Proceeding from simple to complex.—In the teaching of factors, also, the teacher should proeced from simple to complex. In other words, he should start from concrete to abstract. In beginning, certain concrete examples should be taken up. Then abstract factorization may be taught.

Teaching of formula in Algebra.—Formula is a short form of a long statement. It is a method that economize the efforts. It has been defined as below :—

> "It is statement of a general *rule* applicable to anyone of a definite class of problems ; and the statement is expressed in a conventional form for its properties of conciseness and comprehensibility."

Importance of formula.—The knowledge of formula is very important in the teaching of Algebra. It is very helpful in the solution of different problems.

Teaching of graph in Algebra.—In fact, it is a part of Geometry, but it is more common in Algebra also. It has been difined as "a geometrical picture showing arithmetical and algebrical relations."

Need for teaching of graph.—It helps illustration of certain abstract thoughts and figures. Through the graph, it is possible to present things in more concrete manner.

How to teach graph ?—Teaching of graph should not start before class VIII. Graph may be about area, about circle and such other things. The students may be taught to draw graphs neatly and correctly. Unless they are taught to draw it in this manner, its utility shall be exhausted.

Corelation between Algebra, Arithmetic and Geometry.—These are the three branches of Mathemetics and have a strong corelation. Their corelation should be utilised properly. This corelation can make certain concepts more clear. Through

Arithmetic and Geometry, it is possible to make certain Algebraic, concepts clearer. Similarly, through the method of Algebra, it is possible to solve certain complicated sums of Arithmetic as well as Geometry.

POINTS FOR REVISION

1. Algebra is a branch of Mathematics, which saw its origin in Arabia.

2. In this method, difficult problems are solved perforce.

3. Algebra is also called Generalized Mathematics.

4. Following are the aims of the teaching of Algebra :—

(*a*) Removal of ambiguity and deficiencies of language.

(*b*) Helps solution of difficult problems of Arithmetic.

(*c*) Presentation of abstract relationship in new language and new symbols.

(*d*) Simplifies calculation.

(*e*) Provides practical value.

(*f*) Satisfactory checking of results.

(*g*) Encourages use of inductive method.

5. Given below are the causes of the importance of Algebra :—

(*a*) Creates concrete interest in the students.

(*b*) Induces them to study certain topics in a generalised way.

(*c*) Provides better opportunities for study of certain topics.

6. Teaching of Algebra should start at the Junior High School stage of education.

7. In Algebra also exercises and drills occupy an important place.

8. Following are the methods of teaching of Algebra :—

(*a*) Formal method.

(*b*) Equation method.

9. In the teaching of Algebra, certain quantities are expressed in literal numbers.

10. In the teching of Algebra, the following order should be observed in operations :

(*a*) Addition.

(*b*) Substraction.

(*c*) Multiplication.

(*d*) Division.

Negative number can also be added or substracted.

11. Brackets occupy an important place and so the students should be taught about their use. They should also be given the clear idea about the importance of the brackets.

12. Teaching of equation occupies an important place in Algebra. This topic should be taught quite scientifically.

13. The students should be given clear directions about the solution of the problems. They should be told to study the problems seriously and have a clear idea about the underlying meaning.

14. After identical equations, conditional equations should be taken up. After these two, Linear and Quadratic Equations should be taught.

15. In Algebra, numbers are of two types :—

(*a*) Directed numbers.

(*b*) Negative numbers.

16. In the teaching of Algebra, the students should be properly trained in the manipulation of signs of "and".

17. Firstly, addition should be taught in Algebra, then substraction should be taught ; after this, multiplication and then division.

18. Factors occupy an important place. This topic should be taught very scientifically and in an elaborate manner.

19. The teaching of formula should be taken up at a later stage.

20. In fact, formula occupy an important place in the subject matter of Algebra.

21. Algebra is alse taught through graph method. This should begin at a later stage in education, say in class VIII or in higher secondary classes.

22. Algebra is corelated with Arithmetic and Geometry and it should be taught in that manner.

CHAPTER XIX

TEACHING OF GEOMETRY

Q. "A solid foundation of geometrical notions can be laid by practical work." Discuss this statement.

Or

What is Geometry ? How should it be taught and what things should be taught at what stages ? Discuss.

Answer. What is Geometry ?—It is a combination of two words—Geo+metry. Geometry means measurement of the earth. It is informal as well as formal. It has a two-fold value. On the one hand, it provides knowledge and, on the other, it enables the students to do things logically and to develop logical thinking without wich geometry can not be understood.

Importance of the teaching of Geometry.—Teaching of Geometry has several reasons for its importance. Given below are certain causes of teaching and studying Geometry :—

1. It brings about cultural development and is key to mathematical thinking.
2. It has a demonstrative value and develops the power of reasoning.
3. It provides opportunity for observation and exercising the process of deductive logic.
4. It is helpful in providing career to the students.
5. It is also helpful in providing knowledge about certain aspects of life.

Beginning of teaching of Geometry.—Teaching of Geometry should not be taken up before Junior High School stage of education. Normally, it should begin in class VII and go up to higher secondary stage of education. At the higher stage of education, there may be more of advanced Geometry.

Teacher should proceed from simple to complex and from concrete to abstract. Simple exercises in Geometry should be taken up in the beginning and then more difficult ones should be taught.

Stages of the teaching of Geometry.—Teaching of Geometry has been divided into the following three stages :—

1. Experimental stage.
2. Deductive stage.
3. Systematic stage.

Division of these three stages has been done from the point of view of content material, the subject matter and the psychological requirements of the students. In the following lines, an attempt has been made to talk a bit in detail about these stages.

Experimental stage of the teaching of Geometry.—This stage begins from class V and goes on up to class VII. During this period, the student should be taught to recognise figures and draw simple figures and measure them. As far as possible, the teaching of Geometry should be linked with the day-to-day life. They should be provided proper motivation. This stage of teaching of Geometry should stop at the age of 12 or 13.

At this stage of education, attempt should be made to provide knowledge to the students through observation, recognition, inspection, seeing etc. An attempt should also be made to lay foundations for the study of more advanced Geometry or demonstrative geometry at the deductive stage.

Deductive stage of the teaching of Geometry.—This stage should start from class VIII and should end by class IX. As far as possible, students after the age of 15 should not be taught Geometry through the methods suitable for this stage of education.

At this stage of education, several theorems that are fit from the psychological point of view for the students of this stage may be taught. This is, in fact, a period that bridges the gap between the experimental stage of the teaching of Geometry and the systematic stage of the teaching of Geometry.

Systematic stage of Geometry.—This stage of the teaching of Geometry, from the point of view of education, is the higher secondary stage of education. At this stage, the teaching of Geometry should be done in a logical and systematic manner. Pupils should be trained in proper reasoning and thinking. They should be trained to prove things and they should also be given proper practice in logical reasoning. In fact, it is this stage when the major part of the teaching of Geometry is done. This stage should end when the students attain the age of 17 or so.

Aspects of the teaching of Geometry.—Apart from the three stages enumerated above, the teaching of Geometry may also be divided, from the point of view of the subject, under the following two heads :—

(*a*) Modern Geometry, that shall include Conic section and some part of solid Geometry.

(*b*) The philosophy of Geometry. Under this head, the philosophical or non-euclidean aspects of Geometry should be explained.

Different topics of Geometry.—Teaching of Geometry includes several topics on which various theorems, problems and corollaries are based. In the following lines, an attempt should be made to explain the teaching of different aspects of Geometry in a chronological order.

Scale drawing.—Once the students have been taught about the use of the different instrument of Geometry, such as Campass, Protector, etc. they should be taught to draw scales.

Importance of neatness and accuracy.—Before, starting the teaching of Geometry, the importance of neatness and accuracy in the subject should be explained to the pupils. They should also be encouraged to draw lines, angles, etc. accurately. If they learn about these things they shall be able to learn Geometry in a correct and proper perspective.

Angles.—Students should, first of all, be told about angles. Then, they should be asked to draw angles and measure their degrees beginning from straight angle and then the right angle and so on. Once they have learnt about it, they should be told about different types and kinds of angles such as rightangle, acute angle, obtuse angle etc.

This is, in fact, one of the basic concepts of Geometry and it should be properly explained.

Parallel lines.—Once the students have been told about the concept of line, they may be taught parallel lines. This may be done with the help of certain concrete examples of the rails, the lines of the answer book etc.

In the beginning, it is not necessary to explain the definition. It may be taken up at a later stage. What is actually needed is understanding of the concept of parallel lines.

The students should be asked to draw parallel lines on their copy books with the help of the scale and setsquare. Then, they may be taught about the angles drawn by Vivisection.

Congruency.—Knowledge about cougruency may be imparted with the help of the pieces of paper or such other objects. Triangles may be drawn and then on the basis of those triangles, congruency may be taught.

Once it has been made clear, definition may be taken up.

Similarity.—There may be similarity between different figures. It should be explained to the students that all the three angles of a triangle are equal to two rightangles. These angles determine the shape, but it is not possible to determine the size.

In the beginning, things should be made clear with the help of certain concrete examples. Once the concept has been made clear, definitions and other such things should be taken up. Then the practical utility of these things in life may be taught.

Locus.—It is an important concept of Geometry that makes many things clear. It deals with the circle and has practical utility in regard to doors, walls, etc. While teaching about Locus, the students should first be given certain concrete examples and then the problems having bearing on this concept.

Teaching of theorems and problems.—While teaching about theorems and problems, the teacher should proceed from simple to complex. In the beginning, theorems about angles and parallel lines should be taken up and then theorems about rightangles, figures and such other things may be taken up.

Teaching of Problems.—While teaching about problems, the students should be trained to draw a rough figure and do the rough work. Once it has been done, it would be possible to find out the solution.

The students should be taught about the system of writing the proof or solution of the theorems or the problems. In the beginning, the students should be asked to give an analysis and then they should be asked to put it into black and white. After these theorems and the problems, the corollaries may be taken up.

Theorem of Pythogorous.—This is an important theorem of Geometry that has bearing on several other theorems. This should be taught after a good deal of concepts of Geometry have been made clear to the students. In other words, it means that it should be taught at the higher secondary stage of education.

Teaching of definition.—Students should be taught the definition of several things not at the initial stage but when certain concepts have been made clear. In other words, definition should not begin before class VII. What is basically needed is the understanding of the concept.

Theorems about figures having more than three sides :—

These theorems may be about Quadrilateral, Polygon, etc. These things should be taught when the teaching of theorems about triangle has been done.

Teaching of construction.—In Geometry, the problems are solved with the help of construction. In Geometry, it means drawing accurately. Students should be taught about construction only at the Junior High School stage of education, *i. e.*, class VIII. It should continue up to High School or higher secondary stage of education.

Teaching of Proposition.—In Geometry, teaching of proposition is also important. In fact, proposition should be taught at a later stage. In the beginning, the elementary proposition should be made clear and then complicated ones should be taken up. While teaching proposition, the teacher should be guided by his experience in the teaching of Geometry. It is not possible to lay down any set method in this regard.

Certain requisites for the teaching of Geometry.—While teaching Geometry, the following things should be kept in mind. If they are properly followed, the work shall become quite easy :—

1. **Proper symbolic notation.**—The students should be taught about it from the very beginning. Proper practice should be given to them in this regard. If it is done, they should learn things properly and then the whole concept should be clear to them. It is helpful throughout the study of Mathematics. It also saves labour and helps comprehension and retention.

2. Students should also be taught to *translate the exercises into figures*. Figures should be neatly drawn and they should be of a fair size.

3. **Accuracy of language.**—The language that is used should be accurate and the students should be trained in understanding and correct use of the language. If students are able to develop the habit of accuracy of expression, they shall be able to learn the subject quite easily and clearly.

4. **Directions in regard to the theorems.**—*Theorems are quite important in Geometry. Students should be trained in the* following to enable them to have proper understanding of the theorems and to prove them correctly :—

(a) The students should be asked to read the theorem carefully, so that the meaning of the words should be clear to them.

(b) Then, they should be asked to determine the hypothesis and the conclusion.

(c) The figure should then be drawn on the basis of the hypothesis and the letters plotted on the figures as required.

(d) Then what is to be proved should be written.

(e) A method for the proof of the theorem should be selected and then the proof should be written systematically and properly.

POINTS FOR REVISION

1. Geometry is the science of the measurement of earth. Today it is taught as a branch of Mathematics and helps the students to develop logical and accurate thinking.

2. Following are the reasons for the importance of the teaching of Geometry :—

(a) It brings about cultural development.

(b) It is a key to mathematical thinking.

(c) It develops the power of reasoning and has a demonstrative value.

(d) It provides opportunity for observation and exercising the process of deductive logic.

(e) It provides opportunity to the students to take up certain careers and have knowledge about certain aspects of life.

(f) It teaches orderliness and systematic thinking.

3. Teaching of Geometry should begin from class VIII.

4. There are three stages of the teaching of Geometry :—

(a) Experimental stage.

(b) Deductive stage.

(c) Systematic stage.

Experimental stage is between class V and VII and Deductive stage in class VIII and Systematic stage in the higher secondary stage of education.

5. Following are the two aspects of Geometry :—

(*a*) Modern Geometry.

(*b*) Philosophy of Geometry or Philosophical Geometry.

6. Neatness, accuracy and cleanlines occupy an important place in the teaching of Geometry.

7. In the beginning, angles should be taught and then parallel lines be taken up.

8. After these topics, Congruency, Similarity, Locus etc. should be taught.

9. After making these concepts clear, theorems and corollaries should be taken up.

10. Problems should be taught not earlier than class VIII.

11. Once simple theorems have been taught, the Pythagoros theorems should be taught.

12. The teaching of definition should not be done before class VIII.

13. Construction in Geometry should be taught at the highest classes or the Junior High School stage of education.

14. Then the teaching of proposition should be taken up.

15. Proper symbolic notation, translation of exercises into figures, accuracy of language and the direction to the students occupy an important place in the teaching of Geometry.

16. With all these things, the teaching of Geometry may be successfully done.

* * *

CHAPTER XX

TEACHING OF NEW MATHEMATICS

Or

TEACHING OE TRIGONOMETRY, LOGARITHMS, MENSURATION, ETC.

Q. Describe briefly the teaching of new Mathematics at the higher secondary stage of education ?

Or

What methods should be employed in the teaching of Trigonometry, Logarithms, Mensuration, etc. ?

Answer. Introducation of New Mathematics at the higher secondary stage of education—With the new researches in the field of Mathematics, new branches are being introduced in the curriculum of the higher secondary stage of education. It is done with a view to enable the students to grasp higher Mathematics easily and quickly. In fact, the main object is to make the curriculum preparatory as well as terminus. With the introduction of these subjects, it is possible for the students to enter into certain trades. It is also possible for them to prepare themselves for the study of higher Mathematics.

Trigonometry is, in fact, a combination of Arithmetic, Algebra and Geometry. Its teaching should, therefore, be taken up at the higher secondary stage of education. Through Trigonometry, it is possible to help the students to solve certain practical problems. It also provides them with new ideas and new teachniques that may be helpful for Physics, Solid Geometry, etc. It teaches the relations of angles and sides of a triangle.

The New Approach.—The teacning of these subjects is taken up through the following three methods :—

(*a*) The concrete method.

(*b*) The graphical method.

(*c*) The abstract method.

In the beginning the *concrete method* should be employed. Students should be given certain problems having concrete bearing on life. Once the concrete method has gone home, the graphical method should be taken up. Students should be asked to draw graphs of reasonable size. Different problems should be solved through *graph*.

After these two methods, the *abstract method* should be taken up. Through this method, it is possible to solve different problems of Trigonometry. This method has been defined as given below :—

> "This is the traditional method in which the six ratios or at least three of them are defined and their properties developed in a more or less logical sequence. It is not to be recommended except for older pupils, who have reached the age of 15 or more without beginning Trigonometry".

Teaching of Logarithms.—Teaching of this subject helps the students to develop proper attitude. Several problems of Geometry may be solved through this method. There are certain problems for whose solution certain tables have been prepared. Through the help of these tables, it is possible to solve the problems of height, distance, circle, etc.

The teaching of new Mathematics should be done with the object of introducing new techniques to the students. At the higher secondary stage of education, they should not be taught in entirety. They should only be taught in their elementary forms.

POINTS FOR REVISION

1. With recent researches, new topic of Mathematics have been introduced.

2. The new Mathematics includes Trigonometry, Logarithms, etc.

3. The main object of the teaching of these branches of Mathematics is to prepare the students for the study of higher Mathematics.

4. The new Mathematics should be taught in the elementary form.

CHAPTER XXI

LESSON PLAN IN THE TEACHING OF MATHEMATICS

Or

PLANNING OF A LESSON IN THE TEACHING OF MATHEMATICS

Q. Describe briefly the ways of planning a lesson or drawing a lesson plan in the teaching of Mathematics ?

Answer. Planning a lesson.—In education, it is necessary to plan a lesson scientifically. Unless it is done, it is not possible to teach the subject properly. In training colleges, the pupil teachers are, therefore, asked to draw certain lesson plans. On the basis of those lesson plans, they have to teach the classes.

In the teaching of Mathematics, drawing of a lesson plan is equally useful and important. It creates self-confidence in the teacher. It also helps him to know the topic and the students whom he has to teach.

Things to be stated in a lesson plan.—While drawing a lesson plan, the following things are to be put down :—

1. Date.
2. Period.
3. Duration.
4. Class.
5. Name of the institution.
6. Subject.
7. Topic.
8. General objectives.
9. Specific objectives.
10. Previous knowledge of the students.

11. Teaching aids or material aids.
12. Introduction of the topic.
13. Statement of the objects.
14. Presentation of the subject matter.
15. Generalization.
16. Revision or black-board summary.
17. Home task.

Subject and the topic.—Under the subject, the subject to be taught, such as Algebra, Geometry, Arithmrtic, etc. may be mentioned and under the topic, the specific or subject that is to be taught should be mentioned.

General aims.—Under this head, the general aims of the teaching of Mathematics, such as practical aims, disciplinary aims, cultural aims, etc. should be mentioned.

Specific aims or objects.—Under this head, the specific objectives of the teaching of the subject and the topic should be mentioned.

Previous knowledge.—The teacher cannot proceed unless he knows the background of the students about the subject. If the present can be linked and associated with the previous or the past knowledge, it is easy to teach the subject properly. This is the psychological method also.

Material aids or teaching aids.—The teacher uses certain aids and materials to make his teaching effective and interested. These things should be mentioned in the lesson plan.

Introduction.—This is most important part of the lesson plan. It is at this stage that the teacher creates interest of the students in the subject. He has to build the psychology of the students in such a way that they may get interested in the subject. It may be done with the help of the questions and answers which should have bearing on the previous knowledge of the students.

The introduction should be so designed that it is possible for the teacher to create interest and enthusiasm in the students for the subject. This should not take more than 7 or 8 minutes.

Statement of the objects.—Once the students have grown aware of the new subject, the teacher should state the new topic that he is going to introduce before the students. He should stage the object in such a way that it may pose a problem before the students. This statement should be short, definite and attractive.

Presentation.—This may be divided into two parts. In one part the subject or the topic to be taught should be presented in a systematic manner. In the second part the method of teaching may be stated. While writing about the method, the use of the teaching aids should be mentioned.

Generalization.—Once the subject or the topic has become clear to the students, the teacher should try to draw generalizations, with the help of definitions, principles, etc As far as possible, the students should be encouraged to draw generalizations.

Revision.—Once the topic has been given, the teacher should ask certain questions so as to revise the subject. The questions for revision should be pointed.

Black-board summary—After finishing the topic, the teacher should write down the summary of the topic or the lesson on the blackboard. This should be done with the help of the students. Summary should be short and definite.

Home task.—In order to strengthen the knowledge and the experiences, the students should be given some home task. The students should be asked to write out certain things or do some practical experiments etc.

Note.—One very important thing about the lesson is that the teacher should try to finish the lesson within the prescribed limit or within the prescribed period. If he does not do so, he does not do justice to the subject or his job.

Annexed are certain lesson plans on certain topics of Mathematics.

Lesson Plan No. 1

Note.—This is a lesson plan to teach the *area* to the students of *Class VII.*

Date—March 4, 1971. *Period*—Second period. *Duration*—40 Minutes. *Class*—VII.

Institution—G. N. L. D. Rastogi Intermediate College.

subject—Mathematics (Arithmetic). *Topic*—Area.

General objectives.—(1) To develop the power of reasoning of the students and to bring about their intellectual development.

(2) To impart the knowledge of the laws of Mathematics to the students, so that they may use them in their practical life.

(3) To enable the students to face different situations of life, that require the use of Mathematics (Arithmetic).

(4) To develop activity in the students and to develop the capacity in them, to be able to face the difficulties of life.

Specific objective.—To provide knowledge of the area and the rules of area to the students in a clear and scientific manner.

Meterial aids.—*Graph* paper and certain *figures* and *models.*

Previous knowledge.—The students are already acquainted with different types of figures, and they are also aware of the laws of multiplication, division, and other measurements.

They also know about the fields and their boundaries and different forms of the fields.

The students are also aware of elementary principles of Geometry and Algebra.

Introduction.—The teacher may introduce the topic with the help of the following questions :—

1. A field is 15 meters long and 12 meters wide. What shall be the area of this field ?

The students shall come out with the answer.

2. A room is 10 meters long and 8 meters wide. What shall be the area of this room ?

3. The courtyard of a house is 24 meters long and 16 meters wide. How many tiles shall be required to cover the area of the floor if one tile is 1 meter long and 6 meters wide ?

4. Find out the area of a square field whose one side is 25 meters ?

Statement of the objects.—The teacher shall then tell the students that they shall be taught about the area and then he may explain the implications of the questions already asked.

Hypothesis.—He may then explain to the students that the area means the land covered within that figure.

He may then make use of the graph paper and ask the following question :—

Suppose one unit of the graph is equal to 10 centimeter. How many divisions of one centimeter each shall a piece of paper contain, *i. e.*, 50 centimeters long and one centimeter wide. The students should be asked to count those divisions.

Note. The students shall then count the divisions.

They may then be asked to count the divisions lengthwise and breadthwise and then by adding all these divisions, the area of the figures may be found out. They may then be told that the area of the figure shall be equal to the number of divisions in length × the number of the divisions in width.

Then they shall be told that the area is equal to *length* × *breadth.*

The teaching aid may make the matter more clear.

Presentation of the subject-matter.—The teacher may then introduce the subject-matter with the help of another sum or problem.

Example. – A room is 100 meters long and 40 meters wide. How many tiles of 4 meters long and 10 meters wide shall be required to cover the floor of that room ? What shall be the cost of the floor if each tile costs Rs. 10/-.

Presentation :—

1. (1) What is the length of the room ? 100 meters.

(2) What is the breadth of the room ? 40 meters.

(3) What shall be the area of the room ? Length × Breadth

or $100 \times 40 = 4000$ Square meters.

(4) What shall be the area of the floor of the room ? 4000 square meters.

2. (1) What shall be the area of one tile ? $4 \times 10 = 40$ Square meters.

(2) How many tiles shall be required to cover the floor ?

$$\frac{\text{Area of the room}}{\text{Area of the tile}}$$

or $\frac{4000}{40} = 100$

In other words, 100 tiles shall be required to cover that floor.

3. How to find out price ?

One tile costs Rs. 10/-. Therefore, 100 tiles shall cost……… $100 \times 10 =$ Rs.1000/-.

In other words, Rs. 1000/- shall be spent to cover the floor of the room.

Black-board summary.—The teacher then, with the help of the students, may put down the following as the summary on the black-board :—

Area = Length × Breadth,

Similarly, the area of the floor of the room = Length × Breadth.

Similarly, the area of the tile could be found out by multiplying the length by breadth.

Revision.—The teacher may put the following questions to the students and ask them to solve them orally :—

1. A room is 10 meters long and 6 meters wide. What shall be the area of the room ?

2. A playground is 100 meters long and 50 meters wide. What shall be the area of that playground ?

3. How many pieces of matting shall be required to cover the floor of a room whose length is 10 meters × 20 meters while each piece of mat is 2 meters × 5 meters.

Home task.—The teacher may ask the students to solve five sums of area. He may specifically give those sums from the prescribed book or from some other book.

Inspection and the correction work.—The teacher may also ask the students to solve certain sums in the class. While they are doing the sums, the teacher should inspect their work.

The teacher may do the correction work either in the class or he may take home the answer books and correct them. The mistakes of the students should be corrected thoroughly and properly.

Lesson Plan No. 2

Note. This is a lesson plan to teach the factors and their procedure of writing to the students of class IX.

Date—August 11, 1971. *Period*—Second period. *Duration*—40 Minutes. *Class IX.*

Institution—Government Higher Secondary School, Deoria.

Subject—Algebra Topic—*Factorization.*

General objectives.—(1) To develop in the students the power of comprehension about Algebra and its general rules.

(2) To develop in the students the power of reasoning and the time sense.

(3) To develop in the students the interest for the study of Mathematics and Algebra as a branch of Mathematics.

(4) To develop in the students the capacity to solve the problem of future life.

Specific objective.—To teach the students about factorization and other rules of multiplication. They may also be taught the method of factorization and find out multiples.

Material aids.—For this topic or subject, no material aid is required. The students should first be explained the general principles and then asked to do the sums in the practical manner.

Previous knowledge.—(1) The students are already aware of the process of multiplication.

(2) They have also learnt about division and multiplication. Since the students are aware of the fractions and divisions, it shall not be difficult to explain the factorization to them.

Introduction.—The teacher may introduce the topic with the help of the following questions and propositions :—

1. In how many parts 15 may be divided ? 5×3.
2. What is meant by multiple ? When certain parts are multiplied, the result is multiple.
3. What shall be the factorization of $(x+y)^2=(x^2+2\times y+y^2)$
4. $(x^2-y^2)=(x+y)\ (x-y)$
5. $(x-y)^2=(x^2-2\times x+y^2)$

Notes.—Then the students should be told about these principles in detail and in a proper manner.

Statement of the objects or statement of the teacher— The teacher should then say, "Today I shall teach you about Factorization."

Presentation.—He may do so by presenting the following problems before the students :—

$a^2b^2=(ab^2)\times(ab^2)$

In the process of factorization, it shall come to

$(a+b)\ a-b=a^2-ab-b^2+ab=a^2-b^2$

Similarly, $(a+b)^2=(a^2+2ab+b^2)$

$(a-b)^2=(a^2-2ab+b^2)$

For example a^2-5a+b

After giving this example, the students should be asked to find out two digids or numbers whose addition shall come to $5a$ and whose multiple shall be 4. These figures shall be 1 and 4. Therefore, the above mentioned problems shall work out as given below :—

$=a^2-4a-a+4$———1.

$=a(a-4)-1\ (a-4)$——2.

$=(a-4)\ (a-4)$———3.

Then the things should be written and explained to the students in the following manner :—

1. What shall be the factors of $-5a$ in the first step of the question $-4a-a$?

2. Now what shall be the form of the question ? $(a^2-4a-a+4)$

3. Why has it been done so ? (In order to facilitate the factorization.)

4. Now what are the two groups out of which the common factor or quotient may be found out ? a^2-4a and $-a+4$

5. What shall be the common factor or quotient of the first group ? (a)

6. What is the remainder of the first group ? $a(a-4)$

7. What common factor or quotient may be taken out from the second group so that the remaining or remainder may be equal to one factor of the first group ? $-1(a-4)$

8. What shall be the second step of the question now ?

$[a(a-4)-1\ (a-4)]$

9. What shall now be the two factors ? $(a-4)$, $a-1$

Note.—This is a complicated method. The digids and the numbers are large and so it is not possible for the students to understand the factorization. For example,

$$8b^2+126b+55=8b^2+51b+55$$

1. What thing is to be done first in this direction. Find out the factors of 51 ?

2. What should be the relationship of these factors ? (51 should be divided into such parts that their sum or addition may be 51 and their multiple may be $55\times8=440$

3. What are those numbers or digids whose multiple is 440 or addition 55 ? $40+11$

4. What shall now be the new form of question number 1 ? $(8b^2+40a+11a+55)$

5. What are the required groups now ? $(8b^2+40b)$ and $(11a+55)$

6. What common factor should be taken out from first group ? $(8b)$

7. What shall be the remainder ? $(b+5)$

8. What shall then be the form of the first group ? $8(b+5)$

9. Similarly, what is to be seen in the second group while taking out the common factor ? $(b\times5)$ should be the remainder after taking out the common factor.

10. What shall be its form ? $11(b+5)$

11. What shall then be the new form of the whole question ? $\{8b(b+5)+11(a+5)\}$

12. What is the common factor of the two groups that is to be taken out ? $(b+5)$

13. What shall be the remainder of the two groups ? $(8b+11)$

14. What shall now be the form of the question ?

$(b+5)\,(8b+11)$

15. What are the two multiples or factors ? $(b+5), (8b+5$

Blackboard summary.—The teacher may then, with the help of the students, put down the following on the blackboard :—

$8b^2+51b+55$

$=8b^2+40b+11b+55$

$=8b(b+5)+11(b+5)$

$=(+5),(8b+11)$

$=(b+5),(8b+11)$——This shall be the answer.

Revision and home task.—The teacher may give the following question to the students for the revision work and to test the knowledge of the students :—

1. $3a^2+18a+24$
2. $6b^2\div 95b+75$
3. $72a^2-145b+72$
4. $24a^2+76a+40$
5. $56a^2+8a-30$.

Inspection and the correction of the work.—While the students are doing the class work, the teacher should go round and inspect the work. He may continue to do the correction along with it. The remaining answer books he may take home and correct them.

Lesson Plan No. 3.

Note.—This is a lesson plan for teaching a particular theorem of Geometry to the students.

Date—August 10, 1972 *Period*—Second. *Duration*—40 Minutes. *Class*—IX.

Institution—Kisan Intermediate College, Basti.

Subject—Mathematics (Geometry).

Topic—A theorem about parallel lines.

If in a triangle ABC the sides AB and AC have their mid-points as D and E and if they are joined, it is to be proved that DE is parallel and one half of BC.

General objectives. (1) To develop the power of reasoning of the students.

(2) To acquaint the students with the general principles of Geometry and explain their utility.

(3) To activize the intellect of the students and to provide them with the inspiration to solve different problems.

(4) To broaden their outlook and explain the practical utility of different principles.

(5) To develop the power of concentration of the students.

Specific objective.—To teach the students the method of proving certain theorems.

Previous Knowledge.—The students possess the knowledge about the following :—

(*a*) That the lines joining the parallel and equal lines are parallel and equal.

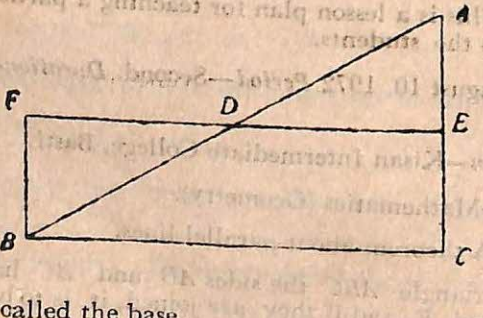
(*b*) They also know if the side angles are equal, the lines are also equal.

(*c*) They are also aware of general definitions and know the things and the principles that are required for explaining all these things.

Introduction.—The teacher may introduce the topic with the help of the following questions :—

1. **How many sides it is a triangle has ? (3 sides).**

2. **What shall be the name of the side BC of the figure given below :—**



It shall be called the base.

3. Why is it that construction is done in order to prove certain propositions ? (In order to make the figures in a position that the things may be proved).

4. What things are to be kept in mind when a thing is to be proved ? (It is quite possible that the students may not be able to furnish a satisfactory answer).

Statement of the objectives.—The teacher shall then state before the students :—

The middle points of the sides *BC* and *AC* of a triangle *ABC* are joined. These are *D* and *E*. Now it is to be proved that the line *DE* is parallel to *BC* and half of it. This shall be the topic for the study.

Presentation of the topic.—(1) In this triangle *ABC* what relationship is to be established between *DE* and *BE* ? (*DE* is parallel and half of *BC*. In other words, *BC* is equal to 2 *DE*).

(2) In order to prove *DE* parallel to *BC*, what shall have to be done ?

Note.—The teacher should explain to the students that some construction will have to be done, so that *BCEF* is formed into a rectangle.

Construction.—How much should *DE* be lengthened in the same direction ?

DE should be drawn up to point *F*, so that *FD* is equal to *DE*.

What should now be done ?

F and *B* should be joined.

What shall be the situation now ?

(*FBCE* is formed into a triangle).

Proof.—It may also be found out with the help of the question-answer method :—

1. What should be done in order to prove that *EC* or *AC* is parallel to *FB* ?

   (The side angle *FBD* should be proved equal to *FDB*).

2. What should be done in order to prove this ?

   (Triangle *EBD* should be proved congruent to *DBE*).

3. What should be done in order to prove these two triangles congruent ?

   Two sides of a triangle and their middle angle is equal to the two sides of the other triangle and the angle between the two sides).

4. In the triangle *FDB* and the triangle *DAE*, which side is equal to which side ?

(Side *FD* is equal to *DA*. It can be done by hypothesis or by supposition).

5. Which side is equal to which side and how ?
(*DE* is equal to *DF* by construction).

6. Are the middle angles of the two also equal ?
(Yes).

7. How are they equal ?
Side *AB* and sides *FE* interest on point *D*. Therefore, angle *FDB* is equal to angle *ADE*, being vertically opposite angles.

8. What shall be the result of this ?
Triangle *FDB* is congruent to triangle *DAE*.

9. What shall be the result of this ?
Angle *FBD* is equal to angle *DAE*.

10. What types of angle they are ?
They are alternate angles.

11. Then what is the situation of the lines having alternate angles ?

(They are parallel).

12. Which sides or lines are parallel ?
FB and *AE*.

13. Are they equal also ?
(Yes).

14. *AE* is equal to which line ?
EC.

15. How are they equal ?
E is the middle point of the line *AC*.

16. Now will *FB* shall be equal to *EC*.
(Yes).

17. How ?
(Because both are equal to *AE*).

In other words, *EB* and *FC* are equal and parallel. Therefore, the lines joining the ends of these lines, *i.e.*, *B*, *C*, *E*, and *F*, shall be equal and parallel. In other words, it means that *BC* and *EF* shall be equal and parallel.

Note.—The teacher may now proceed further with the help of the following questions and answers :—

1. How is it that *DF* is equal to *DE* ?
   (By construction).

2. What part is *DF* of the line *EF* ?
   DF is half of *EF* ?

3. But what is the relationship between the lines *EF* and the other line ?
   It has been proved.

4. What shall be the result of this ?
   The line *DE* is half of the line *BC* and parallel to it because line *BC* and *EF* are equal and parallel.

Black-board work.—The teacher may, after the theorem has been proved, put down the following on the black-board with the help of the students :—

1. **Main proposition.**—In a triangle *ABC*, *D* and *E* are the middle points and sides *AC* and *AB* respectively. It has to be proved that *FF* is parallel to *BC* and half of it.

Construction.—The side *ED* should be increased up to point *F*. It should be lengthened equal to *ED*. Then *FB* and *EC* should be joined.

Proof.—The triangle *FBD* is congruent of triangle *DAE*.

Side *BD* is equal to side *DA* (by hypothesis because *D* is the middle point).

DE is equal to *DF* (by construction). Angle *FDB* is equal to vertically opposite angle *EBA*. Therefore, triangle *DFB* is congruent to triangle *DAE*. Therefore, angle *FBD* is equal to the alternate angle *DAE*. Therefore, *FB* is paralled to *AE* or *AC*.

And side *FB* is equal to side *AE* is equal to *EC* (because *E* is the middle point of side *AC*) or *EB* is equal to and paralled to *EC*. Thereof, *BC* is parallel and equal to *EF*.

But because by construction EF is double of FD and DE, therefore DE is also equal and parallel to EF and BC or BC is parallel to BE. In other words, ED is parallel and half of BC.

Home task and exercises. It is possible that the teacher may ask different students to come to the black-board and prove the things on the figure drawn on the black-board.

All the students may be asked to write down the answers on their answer books.

The students may be asked to do certain corollaries.

Note.—The teacher may also ask the students to do certain things at home.

Correction and inspection.—The teacher should see the work done by the students and correct their answer books.